

Is there an inertia due to the supersymmetry

G Ter-Kazarian*

*Byurakan Astrophysical Observatory
Byurakan 378433, Aragatsotn District, Armenia*

We derive a standard Lorentz code (SLC) of motion by exploring rigid double transformations of, so-called, *master space-induced* supersymmetry (MS-SUSY), subject to certain rules. The renormalizable and actually finite flat-space field theories with $N_{max} = 4$ supersymmetries in four dimensions, if only such symmetries are fundamental to nature, yield the possible *extension of Lorentz code* (ELC), at which the SLC violating new physics appears. In the framework of local MS-SUSY, we address the inertial effects. We argue that a space-time deformation of MS is the origin of inertia effects that can be observed by us. We go beyond the hypothesis of locality. This allows to improve the relevant geometrical structures referred to the noninertial frame in Minkowski space for an arbitrary velocities and characteristic acceleration lengths. This framework furnishes justification for the introduction of the *weak* principle of equivalence, i.e., the *universality of free fall*. The implications of the inertia effects in the more general post-Riemannian geometry are briefly discussed.

PACS numbers: 11.30.Pb, 12.60.Jv, 11.30.Cp, 04.65.+e

I. INTRODUCTION

The principle of inertia, whose origin can be traced back to the works developed by Galileo [1] and Newton [2], is one of the fundamental principles of the classical mechanics. This governs the *uniform motion* of a body and describes how it is affected by applied forces. The universality of gravitation and inertia attribute to the geometry but as having a different natures. However, despite the advocated success of general relativity (GR), the problem of inertia stood open and that this is still an unknown exciting problem to be challenged. The inertia effects cannot be in full generality identified with gravity within GR as it was proposed by Einstein in 1918 [3], because there are many experimental controversies to question the validity of such a description, for details see e.g. [4] and references therein. The model discussed in the latter illustrates the problems of inertia effects, but it also hints at a possible solution. We will not be concerned with the actual details of this model here, but only use it as a backdrop to explore first the SLC in a new perspective of rigid double transformations of, so-called, *master space-induced* supersymmetry (MS-SUSY), subject to certain rules. The theories with extended $N_{max} = 4$ supersymmetries, namely $N = 4$ super-Yang-Mills theories, if only such symmetries are fundamental to nature, lead to the model of ELC in case of the apparent violations of SLC, the possible manifestations of which arise in a similar way in all particle sectors. We show that in the ELC-framework the propagation of the superluminal particle could be consistent with causality, and give a justification of forbiddance of Vavilov-Cherenkov radiation/or analog processes in vacuum. However, we must be careful about the physical relevance of the standard theory of extended supersymmetry which does not allow for chiral fermions, and that its spectrum in no way resembles that of the observed in nature [12]. Consequently, in the framework of local MS-SUSY, we address the *accelerated motion*, while, unlike gravitation, a curvature of space-time now arises entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a *fictitious gravitation* which can be globally removed by appropriate coordinate transformations. The only source of graviton and gravitino, therefore, is the acceleration of a particle. This paper is organized as follows. In the next section, we explain our idea of what is the MS. In section 3, we give a hard look at MS. The MS-SUSY is dealt with in section 4. In section 5, the complementary approach is developed where the SLC and ELC are derived. More about the accelerated motion is said in section 6, this time in the presence of the local MS-SUSY. In section 7 we briefly discuss the inertia effects. We go then beyond the hypothesis of locality in subsections A-C. We compute the improved metric and other relevant geometrical structures in noninertial system of arbitrary accelerating and rotating observer in Minkowski space-time. The case of semi-Riemann background space V_4 is studied in subsection D, whereas we give justification for the introduction of the *weak* principle of equivalence (WPE) on the theoretical basis, which establishes the independence of free-fall trajectories of the internal composition and structure of bodies. The implications of the inertial effects in the more general post-Riemannian geometry are briefly discussed in subsection E. The concluding remarks are presented in section 8. We will be brief and often ruthlessly suppress the indices without notice. Unless otherwise stated we take

*Electronic address: gago'50@yahoo.com

natural units, $\hbar = c = 1$.

II. PRELIMINARIES: A FIRST GLANCE AT MS

With regard now to our original question as to the understanding of the physical processes that underly the *motion*, we tackle the problem in the framework of quantum field theory. Let us consider functional integrals for a quantum-mechanical system with one degree of freedom. Denote by $x(t)$ the position operator in the Heisenberg picture, and by $|x, t\rangle$ its eigenstates. The probability amplitude that a particle which was at x at time t will be at point x' at time t' , also called the Schwinger transformation function for these points, is $F(x't'; xt) = \langle x't' | xt \rangle$. For a particle moving through the two infinitesimally closed points of original space, this in somehow or other implies the elementary act consisting of the *annihilation* of a particle at the point x and time t and, subsequently, its *creation* at the point x' and time t' . The particle can move with different velocities which indicates to existence of the intermediate, so-called, *motion* state. Then the *annihilation* of a particle at point x and time t can intuitively be understood as the transition from the initial state $|x, t\rangle$ to the intermediate *motion* state, $|\underline{x}, \underline{t}\rangle$, yet unknown, where $\underline{x}(\underline{t})$ represent atomic element of idealized *motion* point event. Meanwhile, the *creation* of a particle at infinitesimally closed final point x' and time t' means the subsequent transition from the intermediate *motion* state, $|\underline{x}, \underline{t}\rangle$, to the final state, $|x', t'\rangle$. So, the Schwinger transformation function for two infinitesimally closed points is written in terms of annihilation and creation processes of a particle as

$$F(x't'; xt) = \int d\underline{x} \langle x't' | \underline{x}\underline{t} \rangle \langle \underline{x}\underline{t} | xt \rangle. \quad (1)$$

It should be emphasized that since we do not understand the phenomenon of *motion*, then here it must suffice to expect that the state functions $|x, t\rangle$ and $|\underline{x}, \underline{t}\rangle$ are quite different. Therefore, the intermediate *motion* state, $|\underline{x}, \underline{t}\rangle$, can be defined on say *motion* space, \underline{M} , the points $\underline{x}(\underline{t})$ of which are all the *motion* atomic elements, ($\underline{x}(\underline{t}) \in \underline{M}$). To express Schwinger transformation function, F , as a path integral, we divide the finite time interval into $n+1$ intervals: $t = t_0, t_1, \dots, t_{n+1} = t'$; $t_k = t_0 + k\varepsilon$, where ε can be made arbitrarily small by increasing n . In the limit $n \rightarrow \infty$, by virtue of (1), F becomes an operational definition of the path integral. Hence, in general, in addition to background 4D Minkowski space M_4 , also a background *motion* space \underline{M} , or say *master space*, MS ($\equiv \underline{M}$) is required. So, we now conceive of the two different spaces M_4 and MS, where the geometry of MS is a new physical entity, with degrees of freedom and a dynamics of its own. The above example (1) imposes a constraint upon MS that it was embedded in M_4 as an indispensable individual companion to the particle, without relation to the other matter. In going into practical details, we further adopt the model discussed recently in reference [4], which illustrates the problems of inertia effects, but it also hints at a possible solution. In accord, MS is not measurable directly, but it was argued that a deformation (also see [5]) of MS is the origin of inertia effects that can be observed by us. In general case of 3D motion in M_4 , following [4], a flat MS is the 2D Minkowski space \underline{M}_2 (see next section). In deriving the final step, we should compare and contrast the particle states of quantum fields defined on the background spaces M_4 and \underline{M}_2 , forming a basis in the Hilbert space. It is quite clear that the following properties, being the essence of the chain of transformations (1) for the finite time interval, hold:

1. There should be a particular way of going from each point $x_{i-1}(t_{i-1}) \in M_4$ to the intermediate *motion* point $\underline{x}_{i-1}(\underline{t}_{i-1}) \in \underline{M}_2$ and back $x_i(t_i) \in M_4$, such that the net result of each atomic double transformations is as if we had operated with a space-time *translation* on the original space M_4 . So, the symmetry we are looking for must mix the particle quantum states during the motion in order to reproduce the central relationship between the two successive transformations of this symmetry and the generators of space-time translations. Namely, the subsequent operation of two finite transformations will induce a translation in space and time of the states on which they operate.

2. These successive transformations induce in M_4 the inhomogeneous Lorentz group, or Poincaré group, and that a unitary linear transformation $|x, t\rangle \rightarrow U(\Lambda, a)|x, t\rangle$ on vectors in the physical Hilbert space.

Thus, the underlying algebraic structure of this symmetry generators closes with the algebra of *translations* on the original space M_4 in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra, including the generators of translations. The only symmetry possessing such properties is the supersymmetry (SUSY), see e.g. [6]-[23], which is accepted as a legitimate feature of nature, although it has never been experimentally observed. Certainly we now need to modify the standard theory to have MS-SUSY, involving a superspace which is an enlargement of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ by the inclusion of additional fermion coordinates. Thereby an attempt will be made to treat the *uniform motion* of a particle as a complex process of the global (or rigid) MS-SUSY double transformations. Namely a particle undergoes to *an infinite number of successive transitions from M_4 to \underline{M}_2 and back going permanently through fermion-boson transformations*, which can be interpreted as its *creation* and *annihilation* processes occurring in M_4 or \underline{M}_2 . We derive the *Lorentz code* of motion in terms of spinors referred to MS. This allows to introduce the physical finite *time interval* between two events, as integer number of

the *duration time* of atomic double transition of a particle from M_4 and back. While all the particles are living on M_4 , their superpartners can be viewed as living on \underline{M}_2 .

III. A HARD LOOK AT MS

Following [4], we assume that a flat MS is the 2D Minkowski space:

$$\underline{M}_2 = R_{(+)}^1 \oplus R_{(-)}^1. \quad (2)$$

The ingredient 1D-space $R_{\underline{m}}^1$ is spanned by the coordinates $\eta^{\underline{m}}$. The following notational conventions are used throughout this paper: all magnitudes related to the space \underline{M}_2 will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, \dots = (\pm)$ denote the world indices related to \underline{M}_2 . The metric in \underline{M}_2 is

$$\underline{g} = \underline{g}(\underline{e}_{\underline{m}}, \underline{e}_{\underline{n}}) \underline{\vartheta}^{\underline{m}} \otimes \underline{\vartheta}^{\underline{n}}, \quad (3)$$

where $\underline{\vartheta}^{\underline{m}} = d\eta^{\underline{m}}$ is the infinitesimal displacement. The basis $\underline{e}_{\underline{m}}$ at the point of interest in \underline{M}_2 is consisted of the two real *null vectors*:

$$\underline{g}(\underline{e}_{\underline{m}}, \underline{e}_{\underline{n}}) \equiv \langle \underline{e}_{\underline{m}}, \underline{e}_{\underline{n}} \rangle = {}^*o_{\underline{m}\underline{n}}, \quad ({}^*o_{\underline{m}\underline{n}}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (4)$$

The norm, $i\bar{d} \equiv d\hat{\eta}$, given in the basis (4) reads $i\bar{d} = \underline{e}\underline{\vartheta} = \underline{e}_{\underline{m}} \otimes \underline{\vartheta}^{\underline{m}}$, where $i\bar{d}$ is the tautological tensor field of type (1,1), \underline{e} is a shorthand for the collection of the 2-tuplet $(\underline{e}_{(+)}, \underline{e}_{(-)})$, and $\underline{\vartheta} = \begin{pmatrix} \underline{\vartheta}^{(+)} \\ \underline{\vartheta}^{(-)} \end{pmatrix}$. We may equivalently use a temporal $q^0 \in T^1$ and a spatial $q^1 \in R^1$ variables $q^r(q^0, q^1)(r = 0, 1)$, such that

$$\underline{M}_2 = R^1 \oplus T^1. \quad (5)$$

The norm, $i\bar{d}$, now can be rewritten in terms of displacement, dq^r , as

$$i\bar{d} = d\hat{q} = e_0 \otimes dq^0 + e_1 \otimes dq^1, \quad (6)$$

where e_0 and e_1 are, respectively, the temporal and spatial basis vectors:

$$\begin{aligned} e_0 &= \frac{1}{\sqrt{2}} (\underline{e}_{(+)} + \underline{e}_{(-)}), & e_1 &= \frac{1}{\sqrt{2}} (\underline{e}_{(+)} - \underline{e}_{(-)}), \\ \underline{g}(e_r, e_s) &\equiv \langle e_r, e_s \rangle = o_{rs}, & (o_{rs}) &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \quad (7)$$

The \underline{M}_2 -companion is smoothly (injective and continuous) embedded in the M_4 . Suppose the position of the particle in the background M_4 space is specified by the coordinates $x^m(s)$ ($m = 0, 1, 2, 3$)($x^0 = t$) with respect to the axes of the inertial system $S_{(4)}$. Then, a smooth map $f : \underline{M}_2 \rightarrow M_4$ is defined to be an immersion - an embedding which is a function that is a homeomorphism onto its image:

$$q^0 = \frac{1}{\sqrt{2}} (\eta^{(+)} + \eta^{(-)}) = t, \quad q^1 = \frac{1}{\sqrt{2}} (\eta^{(+)} - \eta^{(-)}) = |\vec{x}|. \quad (8)$$

To motivate why is the MS two dimensional, we note that only two dimensional constructions of real *null vectors* (7) are allowed as the basis at given point in MS, which can be embedded in the (3+1)-dimensional spacetime. This theory is mathematically somewhat similar to the more recent membrane theory, so the \underline{M}_2 can be viewed as 2D space living on the 4D world sheet. Given the inertial frame $S_{(4)}$ in M_4 , we may define the corresponding inertial frame $S_{(2)}$ used by the non-accelerated observer for the position q^r of a free particle in flat \underline{M}_2 . Thereby the time axes of the two systems $S_{(2)}$ and S_4 coincide in direction and that the time coordinates are taken the same, $q^0 = t$. For the case at hand,

$$v^{(\pm)} = \frac{d\eta^{(\pm)}}{dq^0} = \frac{1}{\sqrt{2}}(1 \pm v_q), \quad v_q = \frac{dq^1}{dq^0} = |\vec{v}| = \left| \frac{d\vec{x}}{dt} \right|. \quad (9)$$

So the particle may be viewed as moving simultaneously in M_4 and \underline{M}_2 . Hence, given the inertial frames $S_{(4)}$, $S'_{(4)}$, $S''_{(4)}, \dots$ in M_4 , in this manner we may define the corresponding inertial frames $S_{(2)}$, $S'_{(2)}$, $S''_{(2)}, \dots$ in \underline{M}_2 . Suppose the elements of the Hilbert space can be generated by the action of field-valued operators $\phi(x)(\chi(x), A(x))$ ($x \in M_4$),

where $\chi(x)$ is the Weyl fermion and $A(x)$ is the complex scalar bosonic field defined on M_4 , and accordingly, of field-valued operators $\underline{\phi}(\eta)$ ($\underline{\chi}(\eta)$, $\underline{A}(\eta)$) ($\eta \in \underline{M}_2$), where $\underline{\chi}(\eta)$ is the Weyl fermion and $\underline{A}(\eta)$ is the complex scalar bosonic field defined on \underline{M}_2 , on a translationally invariant vacuum:

$$\begin{aligned} |x\rangle &= \phi(x)|0\rangle, & |x_1, x_2\rangle &= \phi(x_1)\phi(x_2)|0\rangle && \text{referring to } M_4, \\ |\eta\rangle &= \underline{\phi}(\eta)|0\rangle, & |\eta_1, \eta_2\rangle &= \underline{\phi}(\eta_1)\underline{\phi}(\eta_2)|0\rangle && \text{referring to } \underline{M}_2, \end{aligned} \quad (10)$$

etc. The displacement of the field takes the form

$$\phi(x_1 + x_2) = e^{ix_2^m P_m} \phi(x_1) e^{-ix_2^m P_m}, \quad \underline{\phi}(\eta_1 + \eta_2) = e^{i\eta_2^m \underline{P}_m} \underline{\phi}(\eta_1) e^{-i\eta_2^m \underline{P}_m}, \quad (11)$$

where $P_m = i\partial_m$ is the generator of translations on quantum fields $\phi(x)$, and $\underline{P}_m = i\partial_m$ is the generator of translations on quantum fields $\underline{\phi}(\eta) \equiv \underline{\phi}(t, q^1)$. According to the embedding map (8), the relation between the fields $\phi(x)$ and $\underline{\phi}(\eta)$ can be given by the a proper orthochronous Lorentz transformation. For a field of spin- \vec{S} , the general transformation law reads

$$\phi'_\alpha(x') = M_\alpha^\beta \phi_\beta(x) = \exp\left(-\frac{1}{2}\theta^{mn}S_{mn}\right)_\alpha^\beta \phi_\beta(x) = \exp\left(-i\vec{\theta} \cdot \vec{S} - i\vec{\zeta} \cdot \vec{K}\right)_\alpha^\beta \phi_\beta(x), \quad (12)$$

where $\vec{\theta}$ is the rotation angle about an axis \vec{n} ($\vec{\theta} \equiv \theta\vec{n}$), and $\vec{\zeta}$ is the boost vector $\vec{\zeta} \equiv \vec{e}_v \cdot \tan h^{-1} \beta$, provided $\vec{e}_v \equiv \vec{v}/|\vec{v}|$, $\beta \equiv |\vec{v}|/c$, $\theta^i \equiv (1/2)\varepsilon^{ijk}\theta_k$ ($i, j, k = 1, 2, 3$), and $\zeta^i \equiv \theta^{i0} = -\theta^{0i}$. The antisymmetric tensor $S_{mn} = -S_{nm}$, satisfying the commutation relations of the $SL(2, C)$, is the (finite-dimensional) irreducible matrix representations of the Lie algebra of the Lorentz group, and α and β label the components of the matrix representation space, the dimension of which is related to the spin $S^i \equiv (1/2)\varepsilon^{ijk}S_k$ of the particle. The spin \vec{S} generates three-dimensional rotations in space and the $K^i \equiv S^{0i}$ generate the Lorentz-boosts. The fields of spin-zero ($\vec{S} = \vec{K} = 0$) scalar field $A(x)$ and spin-one $A^n(x)$, corresponding to the $(1/2, 1/2)$ representation, transform under a general Lorentz transformation as

$$\underline{A}(\eta) = A(x), \quad \text{spin } 0; \quad \underline{A}^m(\eta) = \Lambda_n^m A^n(x), \quad \text{spin } 1, \quad (13)$$

where the Lorentz transformation is written as

$$\Lambda_n^m(M) \equiv \frac{1}{2} \text{Tr}(\sigma_m M \sigma_n M^\dagger), \quad (14)$$

provided, $\sigma^m \equiv (I_2, \vec{\sigma})$, $\vec{\sigma}$ are Pauli spin matrices. A two-component $(1/2, 0)$ Weyl fermion $\chi_\beta(x)$ transforms under Lorentz transformation, in accord to embedding map (8), as

$$\chi_\beta(x) \longrightarrow \underline{\chi}_\alpha(\eta) = (M_R)_\alpha^\beta \chi_\beta(x), \quad \alpha, \beta = 1, 2 \quad (15)$$

where the rotation matrix is given as

$$M_R = e^{i\frac{1}{2}\sigma_2\theta_2} e^{i\frac{1}{2}\sigma_3\theta_3}. \quad (16)$$

The matrix M_R corresponds to the rotation of an hermitian 2×2 matrix $p^n \sigma_n$:

$$p_q^m \sigma_m = M_R p^n \sigma_n M_R^\dagger, \quad (17)$$

by the angles θ_3 and θ_2 about the axes n_3 and n_2 , respectively, where the standard momentum is $p^n \equiv m(ch\beta, sh\beta \sin \theta_2 \cos \theta_3, sh\beta \sin \theta_2 \sin \theta_3, sh\beta \cos \theta_2)$, and p_q^m is $p_q^m \equiv m(ch\beta, 0, 0, sh\beta)$. According to (15), a two-component $(0, 1/2)$ Weyl spinor field is denoted by $\bar{\chi}^{\dot{\beta}}(x)$, and transforms as

$$\bar{\chi}^{\dot{\beta}}(x) \longrightarrow \bar{\chi}^{\dot{\alpha}}(\eta) = (M_R^{-1})^{\dagger\dot{\alpha}}_{\dot{\beta}} \bar{\chi}^{\dot{\beta}}(x), \quad \dot{\alpha}, \dot{\beta} = 1, 2 \quad (18)$$

where we have used $(M^\dagger)^{\dot{\beta}}_{\dot{\alpha}} = (M^*)_{\dot{\alpha}}^{\dot{\beta}}$. The so-called 'dotted' indices have been introduced to distinguish the $(0, 1/2)$ representation from the $(1/2, 0)$ representation. The bar over the spinor is a convention that this is the $(0, 1/2)$ -representation. The infinitesimal Lorentz transformation matrices for the $(1/2, 0)$ and $(0, 1/2)$ representations,

$$M \simeq I_2 - \frac{i}{2}\vec{\theta} \cdot \vec{\sigma} - \frac{1}{2}\vec{\zeta} \cdot \vec{\sigma}, \quad \text{for } (1/2, 0); \quad (M^{-1})^\dagger \simeq I_2 - \frac{i}{2}\vec{\theta} \cdot \vec{\sigma} + \frac{1}{2}\vec{\zeta} \cdot \vec{\sigma}, \quad \text{for } (0, 1/2) \quad (19)$$

give $S^{mn} = \sigma^{mn}$ for the $(1/2, 0)$ representation and $S^{mn} = \bar{\sigma}^{mn}$ for the $(0, 1/2)$ representation, where the bilinear covariants that transform as a Lorentz second-rank tensor read

$$(\sigma^{mn})_\alpha^\beta \equiv \frac{i}{4}(\sigma_{\alpha\dot{\alpha}}^m \bar{\sigma}^{n\dot{\alpha}\beta} - \sigma_{\alpha\dot{\alpha}}^n \bar{\sigma}^{m\dot{\alpha}\beta}), \quad (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\dot{\beta}} \equiv \frac{i}{4}(\bar{\sigma}^{m\dot{\alpha}\alpha} \sigma_{\alpha\dot{\beta}}^n - \bar{\sigma}^{n\dot{\alpha}\alpha} \sigma_{\alpha\dot{\beta}}^m), \quad (20)$$

provided $\bar{\sigma}^m \equiv (I_2; -\vec{\sigma})$, $(\sigma^m)^*_{\alpha\dot{\beta}} = \sigma_{\beta\dot{\alpha}}^m$ and $(\bar{\sigma}^m)^{\dot{\alpha}\beta} = \bar{\sigma}^{m\dot{\beta}\alpha}$.

IV. MS-SUSY

As alluded to above, a *creation* of a particle in \underline{M}_2 means its transition from M_4 to \underline{M}_2 , while an *annihilation* of a particle in \underline{M}_2 means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in M_4 . Since all fermionic and bosonic states, taken together, form a basis in the Hilbert space, the basis vectors in the Hilbert space, therefore, can be written in the form $|\underline{n}_b, n_f\rangle$ or $|n_b, \underline{n}_f\rangle$, where the boson and fermion occupation numbers are n_b or \underline{n}_b ($= 0, 1, \dots, \infty$) and n_f or \underline{n}_f ($= 0, 1$). So, we may construct the quantum operators, $(q^\dagger, \underline{q}^\dagger)$ and (q, \underline{q}) , which replace bosons by fermions and fermions by bosons, respectively,

$$q^\dagger |\underline{n}_b, n_f\rangle \longrightarrow |\underline{n}_b - 1, n_f + 1\rangle, \quad q |\underline{n}_b, n_f\rangle \longrightarrow |\underline{n}_b + 1, n_f - 1\rangle, \quad (21)$$

and that

$$\underline{q}^\dagger |n_b, \underline{n}_f\rangle \longrightarrow |n_b - 1, \underline{n}_f + 1\rangle, \quad \underline{q} |n_b, \underline{n}_f\rangle \longrightarrow |n_b + 1, \underline{n}_f - 1\rangle. \quad (22)$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators q and \underline{q} . Consider two pairs of creation and annihilation operators (b^\dagger, b) and (f^\dagger, f) for bosons and fermions, respectively, referred to the background space M_4 , as well as $(\underline{b}^\dagger, \underline{b})$ and $(\underline{f}^\dagger, \underline{f})$ for bosons and fermions, respectively, related to the background master space \underline{M}_2 . Putting two operators in one $\bar{B} = (\underline{b}$ or $b)$ and $F = (f$ or $\underline{f})$, the canonical quantization rules can be written most elegantly as

$$\begin{aligned} [B, B^\dagger] &= 1; \quad \{F, F^\dagger\} = 1; \quad [B, B] = [B^\dagger, B^\dagger] = \{F, F\} = \\ \{F^\dagger, F^\dagger\} &= [B, F] = [B, F^\dagger] = [B^\dagger, F] = [B^\dagger, F^\dagger] = 0, \end{aligned} \quad (23)$$

where we note that $\delta_{ij}\delta^3(\vec{p}-\vec{p}')$ and $\delta_{ij}\delta^3(\vec{p}_q-\vec{p}'_q)$ are the unit element 1 of the convolution product $*$, and according to embedding map (8) we have $p_q = \pm|\vec{p}|$ and $p'_q = \pm|\vec{p}'|$. The operators q and \underline{q} can be constructed as

$$q^\dagger = q_0 \underline{b} f^\dagger, \quad q = q_0 \underline{b}^\dagger f, \quad \underline{q}^\dagger = q_0 b \underline{f}^\dagger, \quad \underline{q} = q_0 b^\dagger \underline{f}. \quad (24)$$

So, we may refer the action of the supercharge operators q and q^\dagger to the background space M_4 , having applied in the chain of following transformations of fermion χ (accompanied with the auxiliary field F as it will be seen later on) to boson \underline{A} , defined on \underline{M}_2 :

$$\dots \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \dots \quad (25)$$

Respectively, we may refer the action of the supercharge operators \underline{q} and \underline{q}^\dagger to the \underline{M}_2 , having applied in the chain of following transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field \underline{F}) to boson A , defined on the background space M_4 :

$$\dots \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow \dots \quad (26)$$

Written in one notation, $Q = (q$ or $\underline{q})$, the operators (24) become

$$Q = q_0 B^\dagger F = (q \text{ or } \underline{q}), \quad Q^\dagger = q_0 B F^\dagger = (q^\dagger \text{ or } \underline{q}^\dagger). \quad (27)$$

Due to nilpotent fermionic operators $F^2 = (F^\dagger)^2 = 0$, the operators Q and Q^\dagger also are nilpotent: $Q^2 = (Q^\dagger)^2 = 0$. Hence, the quantum system can be described in one notation by the selfadjoint Hamiltonian $\mathcal{H} = (H_q \equiv \{q^\dagger, q\}$ or $H_{\underline{q}} \equiv \{\underline{q}^\dagger, \underline{q}\})$, and that the generators Q and Q^\dagger satisfy an algebra of anticommutation and commutation relations:

$$\mathcal{H} = \{Q^\dagger, Q\} \geq 0; \quad [\mathcal{H}, Q] = [\mathcal{H}, Q^\dagger] = 0. \quad (28)$$

This is a sum of Hamiltonian of bosonic and fermionic noninteracting oscillators, which decouples, for $Q = q$, into

$$H_q = q_0^2 (\underline{b}^\dagger \underline{b} + f^\dagger f) = q_0^2 (\underline{b}^\dagger \underline{b} + \frac{1}{2}) + q_0^2 (f^\dagger f - \frac{1}{2}) \equiv H_{\underline{b}} + H_f, \quad (29)$$

or, for $Q = \underline{q}$, into

$$H_{\underline{q}} = q_0^2 (b^\dagger b + \underline{f}^\dagger \underline{f}) = q_0^2 (b^\dagger b + \frac{1}{2}) + q_0^2 (\underline{f}^\dagger \underline{f} - \frac{1}{2}) \equiv H_b + H_{\underline{f}}, \quad (30)$$

with the corresponding energies:

$$E_q = q_0^2 (\underline{n}_b + \frac{1}{2}) + q_0^2 (n_f - \frac{1}{2}), \quad E_{\underline{q}} = q_0^2 (n_b + \frac{1}{2}) + q_0^2 (\underline{n}_f - \frac{1}{2}). \quad (31)$$

This formalism manifests its practical and technical virtue in the proposed algebra (28), which becomes more clear in a normalization $q_0 = \sqrt{m}$:

$$\{Q^\dagger, Q\} = 2m; \quad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0. \quad (32)$$

The latter has underlying algebraic structure of the superalgebra for massive *one-particle* states in the rest frame of $N = 1$ SUSY theory without central charges, see e.g. [6]-[23]. This is rather technical topic, and it requires care to do correctly. In what follows we only give a brief sketch. The extension of the MS-SUSY superalgebra (32) in general case when $\vec{p} = i\vec{\partial} \neq 0$ in M_4 or $p_q = i\partial_q \neq 0$ in \underline{M}_2 , and assuming that the resulting motion of a particle in M_4 is governed by the Lorentz symmetries, the MS-SUSY algebra can then be summarized as a non-trivial extension of the Poincaré group algebra thus of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to M_4 . Moreover, if there are several spinor generators Q_α^i with $i = 1, \dots, N$ - theory with N -extended supersymmetry, can be written as a graded Lie algebra (GLA) of SUSY field theories, with commuting and anticommuting generators:

$$\begin{aligned} \{Q_\alpha^i, \bar{Q}_{\dot{\alpha}}^j\} &= 2\delta^{ij} \sigma_{\alpha\dot{\alpha}}^{\hat{m}} p_{\hat{m}}; \quad [p_{\hat{m}}, Q_\alpha^i] = [p_{\hat{m}}, \bar{Q}_{\dot{\alpha}}^j] = 0, \\ \{Q_\alpha^i, Q_\beta^j\} &= \{\bar{Q}_{\dot{\alpha}}^i, \bar{Q}_{\dot{\beta}}^j\} = 0; \quad [p_{\hat{m}}, p_{\hat{n}}] = 0. \end{aligned} \quad (33)$$

Here $\sigma^{(\pm)} = (1/2)(\sigma^o \pm \sigma^3)$, and in order to trace a maximal resemblance in outward appearance to the standard SUSY theories, we set one notation $\hat{m} = (m \text{ if } Q = q, \text{ or } \underline{m} \text{ if } Q = \underline{q})$, no sum over \hat{m} , and as before the indices α and $\dot{\alpha}$ go over 1 and 2. So for both supercharges, q and \underline{q} , we get a supersymmetric models, respectively:

$$\begin{aligned} \{q_\alpha^i, \bar{q}_{\dot{\alpha}}^j\} &= 2\delta^{ij} \sigma_{\alpha\dot{\alpha}}^m p_m; \quad [p_m, q_\alpha^i] = [p_m, \bar{q}_{\dot{\alpha}}^j] = 0, \\ \{q_\alpha^i, q_\beta^j\} &= \{\bar{q}_{\dot{\alpha}}^i, \bar{q}_{\dot{\beta}}^j\} = 0; \quad [p_m, p_n] = 0. \end{aligned} \quad (34)$$

and

$$\begin{aligned} \{\underline{q}_\alpha^i, \bar{\underline{q}}_{\dot{\alpha}}^j\} &= 2\delta^{ij} \sigma_{\alpha\dot{\alpha}}^{\underline{m}} p_{\underline{m}}; \quad [p_{\underline{m}}, \underline{q}_\alpha^i] = [p_{\underline{m}}, \bar{\underline{q}}_{\dot{\alpha}}^j] = 0, \\ \{\underline{q}_\alpha^i, \underline{q}_\beta^j\} &= \{\bar{\underline{q}}_{\dot{\alpha}}^i, \bar{\underline{q}}_{\dot{\beta}}^j\} = 0; \quad [p_{\underline{m}}, p_{\underline{n}}] = 0. \end{aligned} \quad (35)$$

For the self-contained arguments, we should emphasize the crucial differences between the MS-induced SUSY and the standard theories as follows:

- 1) The standard theory can be realized only as a spontaneously broken symmetry since the experiments do not show elementary particles to be accompanied by superpartners with different spin but identical mass. The MS-SUSY, in contrary, can only be realized as an *unbroken supersymmetry*.
- 2) In the standard theory, the Q 's operate on the fields defined on the single M_4 space. It is why the result of a Lorentz transformation in M_4 followed by a supersymmetry transformation is different from that when the order of the transformations is reversed [12]. But, in the MS-SUSY theory, the Q 's ((24), (27)) operate on the fields defined on both M_4 and \underline{M}_2 spaces, fulfilling a transition of a particle between these spaces ($M_4 \rightleftharpoons \underline{M}_2$). So after a Lorentz transformation in M_4 followed by a supersymmetry transformation (which, as we shall see below, now results to uniform motion of a particle with initial constant velocity) we have a particle moving with changed constant velocity. We obtain the same result if we reverse the order of the transformations, namely a Lorentz transformation changes the initial velocity and a supersymmetry transformation followed by a Lorentz transformation just keep the uniform motion with the changed velocity.

We shall forbear to write out further the unitary representations of supersymmetry, giving rise to the notion of supermultiplets, as they are so well known. Also, unless otherwise stated we will not discuss the theories with $N > 1$, because it is unlikely that they play any role in low-energy physics.

A. Wess-Zumino model

To obtain a feeling for this model we may consider first example of non-trivial linear representation of the MS-SUSY algebra in analogy of the Wess-Zumino toy model [24], which has $N = 1$ and $s_0 = 0$, and contains two spin states of a massive Majorana spinor $\psi(\chi, \underline{\chi})$ and two complex scalar fields $\mathcal{A}(A, \underline{A})$ and auxiliary fields $\mathcal{F}(F, \underline{F})$, which provide

in supersymmetry theory the fermionic and bosonic degrees of freedom to be equal. This model is instructive because it contains the essential elements of the MS-induced SUSY. Let us first introduce four additional, anticommuting (Grassmann) parameters $\epsilon^\alpha(\xi^\alpha, \underline{\xi}^\alpha)$ and $\bar{\epsilon}^\alpha(\bar{\xi}^\alpha, \bar{\underline{\xi}}^\alpha)$:

$$\{\epsilon^\alpha, \epsilon^\beta\} = \{\bar{\epsilon}^\alpha, \bar{\epsilon}^\beta\} = \{\epsilon^\alpha, \bar{\epsilon}^\beta\} = 0, \quad \{\epsilon^\alpha, Q_\beta\} = \cdots = [p_{\hat{m}}, \epsilon^\alpha] = 0, \quad (36)$$

which allow to write the algebra (33) ($N = 1$) in terms of commutators only:

$$[\epsilon Q, \bar{Q}\bar{\epsilon}] = 2\epsilon\sigma^{\hat{m}}\bar{\epsilon}p_{\hat{m}}, \quad [\epsilon Q, \epsilon Q] = [\bar{Q}\bar{\epsilon}, \bar{Q}\bar{\epsilon}] = [p^{\hat{m}}, \epsilon Q] = [p^{\hat{m}}, \bar{Q}\bar{\epsilon}] = 0. \quad (37)$$

Here we have dropped the indices $\epsilon Q = \epsilon^\alpha Q_\alpha$ and $\bar{Q}\bar{\epsilon} = \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}$. The infinitesimal supersymmetry transformations for $Q = q$ read

$$\begin{aligned} \delta_\xi \underline{A} &= (\xi q + \bar{\xi} \bar{q}) \times \underline{A} = \sqrt{2} \xi \chi, \\ \delta_\xi \chi &= (\xi q + \bar{\xi} \bar{q}) \times \chi = i\sqrt{2} \sigma^{\hat{m}} \bar{\xi} \partial_{\hat{m}} \underline{A} + \sqrt{2} \xi F, \\ \delta_\xi F &= (\xi q + \bar{\xi} \bar{q}) \times F = i\sqrt{2} \bar{\xi} \sigma^{\hat{m}} \partial_{\hat{m}} \chi; \end{aligned} \quad (38)$$

and for $Q = \underline{q}$ are in the form

$$\begin{aligned} \delta_{\underline{\xi}} A &= (\underline{\xi} \underline{q} + \bar{\underline{\xi}} \bar{\underline{q}}) \times A = \sqrt{2} \underline{\xi} \underline{\chi}, \\ \delta_{\underline{\xi}} \underline{\chi} &= (\underline{\xi} \underline{q} + \bar{\underline{\xi}} \bar{\underline{q}}) \times \underline{\chi} = i\sqrt{2} \sigma^{\hat{m}} \bar{\underline{\xi}} \partial_{\hat{m}} A + \sqrt{2} \underline{\xi} F, \\ \delta_{\underline{\xi}} F &= (\underline{\xi} \underline{q} + \bar{\underline{\xi}} \bar{\underline{q}}) \times F = i\sqrt{2} \bar{\underline{\xi}} \sigma^{\hat{m}} \partial_{\hat{m}} \underline{\chi}, \end{aligned} \quad (39)$$

where according to (13) $A = \underline{A}$. The first relation in (33) means that there should be a particular way of going from one subspace (bosonic/fermionic) to the other and back, such that the net result is as if we had operator of translation $p_{\hat{m}}$ on the original subspace. Actually, it can be checked that the supersymmetry transformations close supersymmetry algebra:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \mathcal{A} = -2i(\epsilon_1 \sigma^{\hat{m}} \bar{\epsilon}_2 - \epsilon_2 \sigma^{\hat{m}} \bar{\epsilon}_1) \partial_{\hat{m}} \mathcal{A}, \quad (40)$$

and likewise for ψ and \mathcal{F} . In the framework of MS-SUSY theory, the Wess-Zumino model has the following Lagrangians:

$$\mathcal{L}_{Q=q} = \mathcal{L}_0 + m\mathcal{L}_m, \quad \mathcal{L}_{Q=\underline{q}} = \underline{\mathcal{L}}_0 + m\underline{\mathcal{L}}_m, \quad (41)$$

provided,

$$\begin{aligned} \mathcal{L}_0 &= i\partial_m \bar{\chi} \sigma^m \chi + \underline{A}^* \square \underline{A} + F^* F, & \mathcal{L}_m &= \underline{A} F + \underline{A}^* F^* - \frac{1}{2} \chi \chi - \frac{1}{2} \bar{\chi} \bar{\chi}, \\ \underline{\mathcal{L}}_0 &= i\partial_{\underline{m}} \bar{\underline{\chi}} \sigma^{\underline{m}} \underline{\chi} + A^* \square A + \underline{F}^* \underline{F}, & \underline{\mathcal{L}}_{\underline{m}} &= A \underline{F} + A^* \underline{F}^* - \frac{1}{2} \underline{\chi} \underline{\chi} - \frac{1}{2} \bar{\underline{\chi}} \bar{\underline{\chi}}, \end{aligned} \quad (42)$$

where according to the embedding map (8), $\square = \underline{\square}$ and $A = \underline{A}$. Whereupon, the equations of motion for the Weyl spinor ψ and complex scalar \mathcal{A} of the same mass m , are

$$\begin{aligned} i\bar{\sigma}^m \partial_m \chi + m\bar{\chi} &= 0, & i\bar{\sigma}^{\underline{m}} \partial_{\underline{m}} \underline{\chi} + m\bar{\underline{\chi}} &= 0, \\ F + m\underline{A}^* &= 0, & \underline{F} + m\underline{A}^* &= 0, \\ \square \underline{A} + mF^* &= 0, & \underline{\square} A + m\underline{F}^* &= 0. \end{aligned} \quad (43)$$

(a) (b)

By virtue of (25) and (26), respectively, (a) stands for $Q = q$ (referring to the motion of a fermion, χ , in M_4) and (b) stands for $Q = \underline{q}$ (so, of a boson, A , in M_4). Finally, the algebraic auxiliary field \mathcal{F} can be eliminated to find

$$\begin{aligned} \mathcal{L}_{Q=q} &= i\partial_m \bar{\chi} \sigma^m \chi - \frac{1}{2} (\chi \chi + \bar{\chi} \bar{\chi}) + \underline{A}^* \square \underline{A} - m^2 \underline{A}^* \underline{A}, \\ \underline{\mathcal{L}}_{Q=\underline{q}} &= i\partial_{\underline{m}} \bar{\underline{\chi}} \sigma^{\underline{m}} \underline{\chi} - \frac{1}{2} (\underline{\chi} \underline{\chi} + \bar{\underline{\chi}} \bar{\underline{\chi}}) + A^* \square A - m^2 A^* A. \end{aligned} \quad (44)$$

B. General superfields

In the framework of standard generalization of the coset construction [25]-[30], we will take $G = G_q \times G_{\underline{q}}$ to be the supergroup generated by the MS-SUSY algebra (33). Let the stability group $H = H_q \times H_{\underline{q}}$ be the Lorentz group (as

to M_4 and \underline{M}_2), and we choose to keep all of G unbroken. Given G and H , we can construct the coset, G/H , by an equivalence relation on the elements of G : $\Omega \sim \Omega h$, where $\Omega = \Omega_q \times \Omega_{\underline{q}} \in G$ and $h = h_q \times h_{\underline{q}} \in H$, so that the coset can be pictured as a section of a fiber bundle with total space, G , and fiber, H . So, the Maurer-Cartan form, $\Omega^{-1}d\Omega$, is valued in the Lie algebra of G , and transforms as follows under a rigid G transformation,

$$\Omega \longrightarrow g\Omega h^{-1}, \quad \Omega^{-1}d\Omega \longrightarrow h(\Omega^{-1}d\Omega)h^{-1} - dh h^{-1}, \quad (45)$$

with $g \in G$. Also we consider a superspace which is an enlargement of $M_4 \oplus \underline{M}_2$ (spanned by the coordinates $X^{\hat{m}} = (x^m, \eta^{\underline{m}})$ by the inclusion of additional fermion coordinates $\Theta^\alpha = (\theta^\alpha, \underline{\theta}^\alpha)$ and $\bar{\Theta}_{\dot{\alpha}} = (\bar{\theta}_{\dot{\alpha}}, \bar{\underline{\theta}}_{\dot{\alpha}})$, as to (q, \underline{q}) , respectively. But note that the relation between the two spinors θ and $\underline{\theta}$ should be derived further from the embedding map (8) (see next section). These spinors satisfy the following relations:

$$\begin{aligned} \{\Theta^\alpha, \Theta^\beta\} &= \{\bar{\Theta}_{\dot{\alpha}}, \bar{\Theta}_{\dot{\beta}}\} = \{\Theta^\alpha, \bar{\Theta}_{\dot{\beta}}\} = 0, \\ [x^m, \theta^\alpha] &= [x^m, \bar{\theta}_{\dot{\alpha}}] = 0, \quad [\eta^{\underline{m}}, \underline{\theta}^\alpha] = [\eta^{\underline{m}}, \bar{\underline{\theta}}_{\dot{\alpha}}] = 0. \end{aligned} \quad (46)$$

and $\Theta^{\alpha*} = \bar{\Theta}^{\dot{\alpha}}$. Points in superspace are then identified by the generalized coordinates $z^M = (X^{\hat{m}}, \Theta^\alpha, \bar{\Theta}_{\dot{\alpha}})$. In case at hand we have then

$$\Omega(X, \Theta, \bar{\Theta}) = e^{i(-X^{\hat{m}}p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})} = \Omega_q(x, \theta, \bar{\theta}) \times \Omega_{\underline{q}}(\eta, \underline{\theta}, \bar{\underline{\theta}}), \quad (47)$$

where we now imply a summation over \hat{m} , etc., such that

$$\Omega_q(x, \theta, \bar{\theta}) = e^{i(-x^m p_m + \theta^\alpha q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{q}^{\dot{\alpha}})}, \quad \Omega_{\underline{q}}(\eta, \underline{\theta}, \bar{\underline{\theta}}) = e^{i(-\eta^{\underline{m}} p_{\underline{m}} + \underline{\theta}^\alpha \underline{q}_\alpha + \bar{\underline{\theta}}_{\dot{\alpha}} \bar{\underline{q}}^{\dot{\alpha}})}. \quad (48)$$

Supersymmetry transformation will be defined as a translation in superspace, specified by the group element

$$g(0, \epsilon, \bar{\epsilon}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})} = g_q(0, \xi, \bar{\xi}) \times g_{\underline{q}}(0, \underline{\xi}, \bar{\underline{\xi}}) = e^{i(\xi^\alpha q_\alpha + \bar{\xi}_{\dot{\alpha}} \bar{q}^{\dot{\alpha}})} \times e^{i(\underline{\xi}^\alpha \underline{q}_\alpha + \bar{\underline{\xi}}_{\dot{\alpha}} \bar{\underline{q}}^{\dot{\alpha}})}, \quad (49)$$

with corresponding anticommuting parameters $\epsilon = (\xi \text{ or } \underline{\xi})$. To study the effect of supersymmetry transformations (45) and $h = 1$, we consider

$$g(0, \epsilon, \bar{\epsilon}) \Omega(X, \Theta, \bar{\Theta}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})} e^{i(-X^{\hat{m}}p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})}. \quad (50)$$

The multiplication of two successive transformations can be computed with the help of the Baker-Campbell-Hausdorff formula $e^A e^B = e^{A+B+(1/2)[A,B]+\dots}$. Hence the transformation (50) induces the motion

$$g(0, \epsilon, \bar{\epsilon}) \Omega(X^{\hat{m}}, \Theta, \bar{\Theta}) \longrightarrow (X^{\hat{m}} + i\Theta \sigma^{\hat{m}} \bar{\epsilon} - i\epsilon \sigma^{\hat{m}} \bar{\Theta}, \Theta + \epsilon, \bar{\Theta} + \bar{\epsilon}), \quad (51)$$

namely,

$$\begin{aligned} g_q(0, \xi, \bar{\xi}) \Omega_q(x, \theta, \bar{\theta}) &\longrightarrow (x^m + i\theta \sigma^m \bar{\xi} - i\xi \sigma^m \bar{\theta}, \theta + \xi, \bar{\theta} + \bar{\xi}), \\ g_{\underline{q}}(0, \underline{\xi}, \bar{\underline{\xi}}) \Omega_{\underline{q}}(\eta, \underline{\theta}, \bar{\underline{\theta}}) &\longrightarrow (\eta^{\underline{m}} + i\underline{\theta} \sigma^{\underline{m}} \bar{\underline{\xi}} - i\underline{\xi} \sigma^{\underline{m}} \bar{\underline{\theta}}, \underline{\theta} + \underline{\xi}, \bar{\underline{\theta}} + \bar{\underline{\xi}}). \end{aligned} \quad (52)$$

The superfield $\Phi(z^M)$, which has a finite number of terms in its expansion in terms of Θ and $\bar{\Theta}$ owing to their anticommuting property, can be considered as the generator of the various components of the supermultiplets. We will consider only a scalar superfield $\Phi'(z^{M'}) = \Phi(z^M)$, an infinitesimal supersymmetry transformation of which is given as

$$\delta_\epsilon \Phi(z^M) = (\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}) \times \Phi(z^M). \quad (53)$$

Acting on this space of functions, the Q and \bar{Q} can be represented as differential operators:

$$Q_\alpha = \frac{\partial}{\partial \Theta^\alpha} - i\sigma^{\hat{m}}_{\alpha\dot{\alpha}} \bar{\Theta}^{\dot{\alpha}} \partial_{\hat{m}}, \quad \bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\Theta}_{\dot{\alpha}}} - i\Theta^\alpha \sigma^{\hat{m}}_{\alpha\dot{\beta}} \bar{\epsilon}^{\dot{\beta}\dot{\alpha}} \partial_{\hat{m}}, \quad (54)$$

where, as usual, the undotted/dotted spinor indices can be raised and lowered with a two dimensional undotted/dotted ϵ -tensors, and the anticommuting derivatives obey the relations

$$\frac{\partial}{\partial \Theta^\alpha} \Theta^\beta = \delta_\alpha^\beta, \quad \frac{\partial}{\partial \bar{\Theta}_{\dot{\alpha}}} \bar{\Theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad \frac{\partial}{\partial \Theta^\alpha} \bar{\Theta}^{\dot{\beta}} \Theta^\gamma = \delta_\alpha^\gamma \bar{\Theta}^{\dot{\beta}} - \delta_\alpha^{\dot{\beta}} \bar{\Theta}^{\dot{\gamma}}, \quad (55)$$

and similarly for $\bar{\Theta}$. In order to write the exterior product in terms of differential operators, one induces a new basis as

$$e^A(z) = dZ^M e_M^A(z), \quad (56)$$

and that

$$D_A = e_A^N(z) \frac{\partial}{\partial z^N}, \quad (57)$$

where to be brief we left implicit the symbol \wedge in writing of exterior product. The covariant derivative operators

$$D_{\hat{m}} = \partial_{\hat{m}}, \quad D_\alpha = \frac{\partial}{\partial \Theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}^{\hat{m}} \bar{\Theta}^{\dot{\alpha}} \partial_{\hat{m}}, \quad \bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\Theta}_{\dot{\alpha}}} + i\Theta^\alpha \sigma_{\alpha\dot{\beta}}^{\hat{m}} \bar{\epsilon}^{\dot{\beta}\dot{\alpha}} \partial_{\hat{m}}, \quad (58)$$

anticommute with the Q and \bar{Q}

$$\{Q_\alpha, D_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = \{Q_\alpha, \bar{D}_{\dot{\beta}}\} = \{\bar{Q}_{\dot{\alpha}}, D_\beta\} = 0, \quad (59)$$

and satisfy the following structure relations:

$$\{D_\alpha, D_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}^{\hat{m}} \partial_{\hat{m}}, \quad \{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0. \quad (60)$$

From (58), we obtain

$$e_A^M = \begin{pmatrix} e_{\hat{a}}^{\hat{m}} = \delta_{\hat{a}}^{\hat{m}} & e_{\hat{a}}^\mu = 0 & e_{\hat{a}\mu} = 0 \\ e_\alpha^{\hat{m}} = i\sigma_{\alpha\dot{\alpha}}^{\hat{m}} \bar{\Theta}^{\dot{\alpha}} & e_\alpha^\mu = \delta_\alpha^\mu & e_{\alpha\mu} = 0 \\ e^{\dot{\alpha}\hat{m}} = i\Theta^\alpha \sigma_{\alpha\dot{\beta}}^{\hat{m}} \bar{\epsilon}^{\dot{\beta}\dot{\alpha}} & e^{\dot{\alpha}\mu} = 0 & e^{\dot{\alpha}\mu} = \delta_{\dot{\mu}}^{\dot{\alpha}} \end{pmatrix}, \quad (61)$$

where $\hat{a} = (a \text{ or } \underline{a})$, $a = 0, 1, 2, 3$; $\underline{a} = (+), (-)$. The supersymmetry transformations of the component fields can be found using the differential operators (58). The covariant constraint

$$\bar{D}_{\dot{\alpha}} \Phi(z^M) = 0, \quad (62)$$

which does not impose equations of motion on the component fields, defines the chiral superfield, Φ . Under the supersymmetry transformation (51) the chiral field transforms as follows:

$$\delta_\xi \Phi = (\xi q + \bar{\xi} \bar{q}) \times \Phi = \delta_\xi \underline{A}(\eta) + \sqrt{2}\theta \delta_\xi \chi(x) + \theta \theta \delta_\xi F(x) + \dots \quad (63)$$

in case of $Q = q$, and

$$\delta_{\underline{\xi}} \underline{\Phi} = (\underline{\xi} q + \bar{\underline{\xi}} \bar{q}) \times \underline{\Phi} = \delta_{\underline{\xi}} A(x) + \sqrt{2}\theta \delta_{\underline{\xi}} \underline{\chi}(\eta) + \underline{\theta} \underline{\theta} \delta_{\underline{\xi}} \underline{F}(\eta) + \dots \quad (64)$$

in case of $Q = \underline{q}$, where as before $A(x) = \underline{A}(\eta)$, and the supersymmetry transformations are decoupled to (38) and (39). Equations (63) and (64) show that the chiral superfield contains the same component fields as the Wess-Zumino model for MS-SUSY theory. The supervolume integrals of products of superfields constructed in the superspace $(x^m, \theta, \bar{\theta})$ will lead to the supersymmetric kinetic energy for the Wess-Zumino model

$$\int d^4x d^4\theta \Phi^\dagger \Phi, \quad (65)$$

where the superspace Lagrangian reads

$$\Phi^\dagger \Phi = \underline{A}^* \underline{A} + \dots + \theta \theta \bar{\theta} \bar{\theta} [\frac{1}{4} \underline{A}^* \square \underline{A} + \frac{1}{4} \square \underline{A}^* \underline{A} - \frac{1}{2} \partial_m \underline{A}^* \partial^m \underline{A} + F^* F + \frac{i}{2} \partial_m \bar{\chi} \bar{\sigma}^m \chi - \frac{i}{2} \bar{\chi} \bar{\sigma}^m \partial_m \chi], \quad (66)$$

where $\square A = \square \underline{A}$ and $\partial_m \underline{A}^* \partial^m \underline{A} = \partial_m A^* \partial^m A$. Similarly, the supersymmetric kinetic energy for the Wess-Zumino model constructed in the superspace $(\eta^{\underline{m}}, \underline{\theta}, \bar{\underline{\theta}})$ for MS-SUSY theory is

$$\int d^2\eta d^4\underline{\theta} \underline{\Phi}^\dagger \underline{\Phi}, \quad (67)$$

where the superspace Lagrangian is written down

$$\underline{\Phi}^\dagger \underline{\Phi} = A^* A + \dots + \underline{\theta} \underline{\theta} \bar{\underline{\theta}} \bar{\underline{\theta}} [\frac{1}{4} A^* \square A + \frac{1}{4} \square A^* A - \frac{1}{2} \partial_m A^* \partial^m A + \underline{F}^* \underline{F} + \frac{i}{2} \partial_m \bar{\underline{\chi}} \bar{\sigma}^m \underline{\chi} - \frac{i}{2} \bar{\underline{\chi}} \bar{\sigma}^m \partial_m \underline{\chi}]. \quad (68)$$

To complete the model, we also need superspace expressions for the masses and couplings, which can be easily found in analogy of the standard theory, namely: 1) fermion masses and Yukawa couplings, $(\partial^2 \mathcal{P} / \partial \mathcal{A}^2) \psi \psi$; and 2) the scalar potential, $\mathcal{V}(\mathcal{A}, \mathcal{A}^*) = |\partial \mathcal{P} / \partial \mathcal{A}|^2$; where $\mathcal{P} = (1/2) m \Phi^2 + (1/3) \lambda \Phi^3$ is the most general renormalizable interaction for a single chiral superfield. Thereby, the auxiliary field equation of motion reads $\mathcal{F}^* + (\partial \mathcal{P} / \partial \mathcal{A}) = 0$. Similarly, we can treat the vector superfields, etc. Here we shall forbear to write them out as the standard theory is so well known.

V. UNACCELERATED UNIFORM MOTION; A FOUNDATION OF SR

Let impose peculiar constraints upon the anticommuting spinors $(\underline{\xi}, \bar{\underline{\xi}})$ and $(\underline{\xi}, \bar{\underline{\xi}})$:

$$\underline{\xi}^\alpha = i \frac{\tau}{\sqrt{2}} \underline{\theta}^\alpha, \quad \bar{\underline{\xi}}_{\dot{\alpha}} = -i \frac{\tau^*}{\sqrt{2}} \bar{\underline{\theta}}_{\dot{\alpha}}, \quad \xi^\alpha = i \frac{\tau}{\sqrt{2}} \theta^\alpha, \quad \bar{\xi}_{\dot{\alpha}} = -i \frac{\tau^*}{\sqrt{2}} \bar{\theta}_{\dot{\alpha}}, \quad (69)$$

and write down the infinitesimal displacement arisen in \underline{M}_2 as

$$\Delta \eta^{\underline{m}} = v^{\underline{m}} \tau = \underline{\theta} \sigma^{\underline{m}} \bar{\underline{\xi}} - \underline{\xi} \sigma^{\underline{m}} \bar{\underline{\theta}}, \quad (70)$$

where the parameter $\tau (= \tau^*)$ can physically be interpreted as the *duration time* of atomic double transition of a particle from M_4 to \underline{M}_2 and back. So,

$$v^{(+)} \tau = i(\underline{\theta}_1 \bar{\underline{\xi}}_1 - \underline{\xi}_1 \bar{\underline{\theta}}_1), \quad v^{(-)} \tau = i(\underline{\theta}_2 \bar{\underline{\xi}}_2 - \underline{\xi}_2 \bar{\underline{\theta}}_2), \quad (71)$$

and that

$$v^2 \tau^2 = v^{(+)} v^{(-)} \tau^2 = -(\underline{\theta}_1 \bar{\underline{\xi}}_1 - \underline{\xi}_1 \bar{\underline{\theta}}_1)(\underline{\theta}_2 \bar{\underline{\xi}}_2 - \underline{\xi}_2 \bar{\underline{\theta}}_2) = 4 \underline{\theta}_1 \bar{\underline{\theta}}_1 \underline{\theta}_2 \bar{\underline{\theta}}_2 \geq 0. \quad (72)$$

Hence

$$v^{(+)} = \sqrt{2} \underline{\theta}_1 \bar{\underline{\theta}}_1 \geq 0, \quad v^{(-)} = \sqrt{2} \underline{\theta}_2 \bar{\underline{\theta}}_2 \geq 0. \quad (73)$$

According to embedding map (8), therefore, we may introduce the *velocity of light* (c) in vacuum as maximum attainable velocity for uniform motions of all the particles in the Minkowski background space, M_4 :

$$c = \frac{1}{\sqrt{2}}(v^{(+)} + v^{(-)}) = \sqrt{2}(\underline{\theta}_1 \bar{\underline{\theta}}_1 + \underline{\theta}_2 \bar{\underline{\theta}}_2) = \sqrt{2} \underline{\theta} \bar{\underline{\theta}} = \text{const}, \quad (74)$$

$$v_q = \frac{1}{\sqrt{2}}(v^{(+)} - v^{(-)}) = \sqrt{2}(\underline{\theta}_1 \bar{\underline{\theta}}_1 - \underline{\theta}_2 \bar{\underline{\theta}}_2) = \pm |\vec{v}| = \text{const}, \quad |\vec{v}| \leq c.$$

The spinors $\underline{\theta}(\underline{\theta}, \underline{\xi})$ and $\xi(\underline{\theta}, \underline{\xi})$ satisfy the embedding map (8), namely $\Delta q^0 = \Delta x^0$ and $\Delta q^2 = (\Delta \vec{x})^2$, so from (52) we have

$$\underline{\theta} \sigma^0 \bar{\underline{\xi}} - \underline{\xi} \sigma^0 \bar{\underline{\theta}} = \theta \sigma^0 \bar{\xi} - \xi \sigma^0 \bar{\theta}, \quad (\underline{\theta} \sigma^3 \bar{\underline{\xi}} - \underline{\xi} \sigma^3 \bar{\underline{\theta}})^2 = (\theta \vec{\sigma} \bar{\xi} - \xi \vec{\sigma} \bar{\theta})^2. \quad (75)$$

By virtue of (69), the (75) reduces to

$$\underline{\theta}_1 \bar{\underline{\theta}}_1 + \underline{\theta}_2 \bar{\underline{\theta}}_2 = \underline{\theta} \bar{\underline{\theta}} = \theta \bar{\theta}, \quad \underline{\theta}_1 \bar{\underline{\theta}}_1 - \underline{\theta}_2 \bar{\underline{\theta}}_2 = \pm \sqrt{\frac{3}{2}}(-\theta \theta \bar{\theta} \bar{\theta})^{1/2} = \pm \sqrt{\frac{3}{2}} \theta \bar{\theta}, \quad (76)$$

where we use the following relations:

$$(\theta \sigma^m \bar{\theta})(\theta \sigma^n \bar{\theta}) = \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} \eta^{mn}, \quad (-\theta \theta \bar{\theta} \bar{\theta})^{1/2} = (\theta \bar{\theta} \theta \bar{\theta})^{1/2} = \theta \bar{\theta}. \quad (77)$$

So,

$$\underline{\theta}_1 \bar{\underline{\theta}}_1 = \frac{1}{2}(1 \pm \sqrt{\frac{3}{2}}) \theta \bar{\theta}, \quad \underline{\theta}_2 \bar{\underline{\theta}}_2 = \frac{1}{2}(1 \mp \sqrt{\frac{3}{2}}) \theta \bar{\theta}. \quad (78)$$

Hence we conclude that the *unaccelerated uniform motion of a particle in M_4 is encoded in the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to the master space \underline{M}_2 , which is an individual companion to the particle of interest*. Therefore, to account for the most important two postulates constituting a foundation of SR, it would be necessary further to impose certain constraints upon the constant Lorentz spinors $\underline{\theta}$. Lorentz invariance is a fundamental symmetry and refers to measurements of ideal inertial observers that move uniformly forever on rectilinear timelike worldlines. In view of relativity of velocity of a particle, we are of course not limited to any particular spinor $\underline{\theta}(\vec{v})$, but can choose at will any other spinors $\underline{\theta}'(\vec{v}')$, $\underline{\theta}''(\vec{v}'')$, \dots relating respectively to velocities $\vec{v}', \vec{v}'', \dots$, whose functional dependence (transformational law) on the original spinor $\underline{\theta}(\vec{v})$ is known. Of the various possible transformations, we must consider for a validity of SR only those which obey the following constraints:

$$\begin{aligned} 1. \quad & \underline{\theta} \bar{\underline{\theta}} = \underline{\theta}' \bar{\underline{\theta}}' = \underline{\theta}'' \bar{\underline{\theta}}'' = \dots = \frac{c}{\sqrt{2}} = \text{const}; \\ 2. \quad & \underline{\theta}_1 \bar{\underline{\xi}}_1 \underline{\theta}_2 \bar{\underline{\xi}}_2 = \underline{\theta}'_1 \bar{\underline{\xi}}'_1 \underline{\theta}'_2 \bar{\underline{\xi}}'_2 = \dots = \text{inv}. \end{aligned} \quad (79)$$

According to first relation, we may introduce a notion of *time*, for the all inertial frames of reference S, S', S'', ..., we have then standard Lorentz code (SLC)-relations: $x^0 = ct$, $x^{0'} = ct'$, $x^{0''} = ct''$, ... This is a second postulate of SR (Einstein's postulate) that the velocity of light (c) in free space appears the same to all observers regardless the relative motion of the source of light and the observer. By virtue of second relation and equations (69), we may derive the invariant interval between the two events defined in Minkowski spacetime:

$$\begin{aligned} 8\bar{\theta}_1 \bar{\theta}_1 \theta_2 \bar{\theta}_2 \Delta t^2 &= 2v^2 \Delta t^2 = (c^2 - v_q^2) \Delta t^2 = (c^2 - \bar{v}^2) \Delta t^2 = \\ c^2 \Delta t^2 - \Delta \bar{x}^2 &\equiv \Delta s^2 = 8\bar{\theta}'_1 \bar{\theta}'_1 \theta'_2 \bar{\theta}'_2 \Delta t'^2 = c^2 \Delta t'^2 - \Delta \bar{x}'^2 \equiv \Delta s'^2 = \dots = inv, \end{aligned} \quad (80)$$

where we introduce the physical finite *time interval*, $\Delta t = k\tau$, between two events as integer number of the *duration time*, τ , of atomic double transition of a particle from M_4 to \underline{M}_2 and back, where k is the number of double transformations. Hence, an unaccelerated uniform motion, for example, of spin-0 particle in M_4 can be described by the chiral superfield $\underline{\Phi}(\eta^{\bar{m}}, \underline{\theta}, \bar{\theta})$, while a similar motion of spin-1/2 particle in M_4 can be described by the chiral superfield $\Phi(x^m, \theta, \bar{\theta})$, etc. So, we may refer to all constant Lorentz spinors obeying (79) as the *SLC-spinors*, which constitute a foundation of SR. Hence, in view of the MS-SUSY mechanism of motion, the uniform motion of a particle in M_4 is encoded in the spinors $\underline{\theta}$ and $\bar{\theta}$, which refer to \underline{M}_2 . This will call for a complete reconsideration of our ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property.

A. Extended supersymmetry and ELC

In four dimensions, it is possible to have as many as eight supersymmetries: $N_{max} = 4$ for renormalizable flat-space field theories; $N_{max} = 8$ for consistent theories of supergravity. It has been shown that the $N = 4$ theory is not only renormalizable but actually finite. So, the theories with $N > 1$ may play a key role in high-energy physics. These models explore in (50) more than one distinct copy of the supersymmetry generators, $Q_{\alpha i}$, therefore, this perspective ultimately requires to relax the Einstein's postulate, because it is natural now to circumvent the limitations to any particular spinor $\underline{\theta}$, instead, considering $i(= 1, \dots, 4)$ -th ($N_{max} = 4$) copy of the spinors $\Theta^{\alpha i} \equiv (\theta^{\alpha i}$ or $\underline{\theta}^{\alpha i})$. Therefore, we now have a straightforward generalization of (79):

$$\begin{aligned} 1. \underline{\theta}^i \bar{\theta}^i &= \underline{\theta}^{i'} \bar{\theta}^{i'} = \underline{\theta}^{i''} \bar{\theta}^{i''} = \dots = \frac{c_i}{\sqrt{2}} = const; & (\text{no sum over } i), \\ 2. \underline{\theta}^i \bar{\theta}_1 \bar{\theta}_2 \bar{\theta}_2 \bar{\theta}_1 &= \underline{\theta}^{i'} \bar{\theta}_1 \bar{\theta}_2 \bar{\theta}_2 \bar{\theta}_1 = \dots = inv. \end{aligned} \quad (81)$$

This observation allows us to lay forth the extension of Lorentz code, at which SLC violating new physics appears. We may now consider the particles of $i(= 1, \dots, 4)$ -th *type* ($N_{max} = 4$). That is to say, *the i -th type particle in free Minkowski space carries an individual Lorentz motion code with its own maximum attainable velocity c_i , i.e., its own velocity of 'light-like' state:*

$$\begin{aligned} c_i &= \frac{1}{\sqrt{2}}(v_i^{(+)} + v_i^{(-)}) = \sqrt{2}(\underline{\theta}_{1i} \bar{\theta}_{1i} + \underline{\theta}_{2i} \bar{\theta}_{2i}) = \sqrt{2}\underline{\theta}_i \bar{\theta}_i = const, & (\text{no sum over } i), \\ v_{qi} &= \frac{1}{\sqrt{2}}(v_i^{(+)} - v_i^{(-)}) = \sqrt{2}(\underline{\theta}_{1i} \bar{\theta}_{1i} - \underline{\theta}_{2i} \bar{\theta}_{2i}) = \pm|\vec{v}_i| = const, & |\vec{v}_i| \leq c_i. \end{aligned} \quad (82)$$

A general solution to the Lorentz covariance in the theory can be easily accommodated if the 'time' at which event occurs is extended by allowing an extra dependence on 'different type' readings t_i referred to the particles of different type. They satisfy for all inertial frames of reference S, S', S'', ..., so-called 'ELC-relations':

$$\begin{aligned} x^0 &\equiv c_1 t_1 = \dots = c_i t_i = \dots, \\ x^{0'} &\equiv c_1 t'_1 = \dots = c_i t'_i = \dots, \\ &\dots\dots\dots \end{aligned} \quad (83)$$

where $c_1 \equiv c$ is the speed of light in vacuum, and $c_i > c_1$, ($i = 2, 3, 4$) are speeds of the additional 'light-like' states, higher than that of light. The clock reading t_i can be used for the i -th type particle, the velocity of which reads $v_i = x/t_i = c_i x/x^0$, so $\beta = v_1/c_1 = \dots v_i/c_i = \dots \equiv v/c = x/x^0$. If $v_i = c_i$ then $v_1 = c_1$, and the proper time of 'light-like' states are described by the null vectors $ds_1^2 = \dots ds_i^2 = \dots = 0$. The extended Lorentz transformation equations for given i -th and j -th type clock readings can be written then in the form

$$x' = \gamma(x - vt), \quad t'_i = \gamma \frac{c_i}{c_1} (t_j - \frac{v_j}{c_j^2} x), \quad y' = y, \quad z' = z, \quad \gamma \equiv 1/\sqrt{1 - \beta^2}. \quad (84)$$

Hence, like the standard SR theory, regardless the type of clock, a metre stick traveling with system S measures shorter in the same ratio, when the simultaneous positions of its ends are observed in the other system S': $dx' = dx/\gamma$.

Furthermore, a time interval dt_i specified by the i -th type readings, which occur at the same point in system S ($dx = 0$), will be specified with the j -th type readings of system S' as $dt'_j = \gamma(c_i/c_j)dt_i$. Here we have called attention to the fact that the mere composition of velocities which are not themselves greater than that of c_i will never lead to a speed that is greater than that of c_i . Inevitably in the ELC-framework a specific task is arisen then to distinguish the type of particles. This evidently cannot be done when the velocity ranges of different type particles intersect. To reconcile this situation, we note that, according to (83), we may freely interchange the types of particles in the intersection. Therefore, we adopt following convention. With no loss of generality, we may re-arrange a general solution that the particles with velocities $v_1 < c_1$, regardless their type, will be treated as the 1-th type particles and, thus, a common clock reading for them and light will be set as t_1 . This part of a formalism is completely equivalent to the SLC-framework. Successively, the particles, other than 'light-like' ones, with velocities in the range $c_{i-1} \leq v_i < c_i$, regardless their type, will be treated as the i -th type particles and, thus, a common clock reading for them and 'light-like' state (i) will be set as t_i . The invariant momentum

$$p_i^2 = p_{\mu i} p_i^\mu = \left(\frac{E_i}{c_i}\right)^2 - \vec{p}_i^2 = m_{0i}^2 c_i^2 = p_1^2 = p_{\mu 1} p_1^\mu = \left(\frac{E_1}{c_1}\right)^2 - \vec{p}_1^2 = m_0^2 c_1^2, \quad (85)$$

introduces a *modified dispersion relation* for i -th type particle:

$$E_i^2 = \vec{p}_i^2 c_i^2 + m_{0i}^2 c_i^4 = \vec{p}_i^2 c_i^2 + m_{01}^2 c_1^2 c_i^2, \quad (86)$$

where the mass of i -th type particle has the value m_{0i} , when at rest, the positive energy is

$$E_i = m_i c_i^2 = \gamma m_{0i} c_i^2 = \gamma m_{01} c_1 c_i, \quad (87)$$

and $\vec{p}_i = m_i \vec{v}_i = \gamma m_{0i} \vec{v}_i$ is the momentum. The relation (87) modifies the well-known Einsteins equation that energy E always has immediately associated with it a positive mass $m_i = \gamma m_{0i}$, when moving with the velocity \vec{v}_i . Having set this theoretical background, one may find some consequences for the superluminal propagation of particles. In particular, in the ELC-framework of uniform motion, the time elapsing between the cause t_{iA} and its effect t_{iB} as measured for the i -th type superluminal particle is

$$\Delta t_i = t_{iB} - t_{iA} = \frac{x_B - x_A}{v_i}, \quad (88)$$

where x_A and x_B are the coordinates of the two points A and B. In another system S', which is chosen as before and has the arbitrary velocity $V \equiv V_j$ with respect to S, the time elapsing between cause and effect would be

$$\Delta t'_i = \frac{1 - \frac{V_j}{c_j} \frac{v_i}{c_i}}{\sqrt{1 - \frac{V_j^2}{c_j^2}}} \Delta t_i \geq 0, \quad (89)$$

where according to (81), $t_{iB} = (c_j/c_i)t_{jB}$ and $t_{iA} = (c_j/c_i)t_{jA}$. That is, *the ELC-framework recovers the causality for a superluminal propagation*, so the starting of the superluminal impulse at A and the resulting phenomenon at B are being connected by the relation of cause and effect in arbitrary inertial frames. Furthermore, in this framework, we may give a justification of forbiddance of Vavilov-Cherenkov radiation/or analog processes in vacuum. Thereby, in this framework we have to set, for example, $k_1 = (\frac{\omega}{c_1}, \vec{k}_1)$ for the 1-th type γ_1 photon, provided $\vec{k}_1 = \vec{e}_k \frac{\omega}{c_1}$, and $p_2 = (\frac{E_2}{c_2}, \vec{p}_2)$ for the 2-nd type superluminal particle. Then the process ($l_2 \rightarrow l_2 + \gamma_1$) becomes kinematically permitted if and only if

$$k_1 p_2 = \frac{\omega}{c_1} \frac{E_2}{c_2} \left(1 - \vec{e}_k \frac{\vec{v}_2}{c_2}\right) = 0, \quad (90)$$

which yields $\omega \equiv 0$ because of $\left(1 - \vec{e}_k \frac{\vec{v}_2}{c_2}\right) \neq 0$. This evades constraints due to VC-like processes since the superluminal particle $\nu_{\mu 2}$ does not actually travel faster than the speed c_2 . Finally, in ELC-framework we discuss the VC-radiation of the charged superluminal particle propagating in vacuum with a constant speed $v_2 > c_1$ higher than that of light. Recall that, for a charged particle ($e \neq 0$) moving in a transparent, isotropic and non-magnetic medium with a constant velocity higher than velocity of light in this medium the VC radiation is allowed. The energy loss per frequency is [31]

$$dF = -d\omega \frac{ie^2}{2\pi} \sum \omega \left(\frac{1}{c^2} - \frac{1}{\varepsilon v^2}\right) \int \frac{d\zeta}{\zeta}, \quad (91)$$

where the direction of the velocity \vec{v} is chosen to be x -direction: $k_x = k \cos \theta = \omega/v$, $k = n\omega/c$ is the wave number $n = \sqrt{\varepsilon}$ is the real refractive index, ε is the permittivity. The summation is over terms with $\omega = \pm|\omega|$, and a variable

$$\zeta = q^2 - \omega^2 \left(\frac{\varepsilon}{c^2} - \frac{1}{v^2}\right) \quad (92)$$

is introduced, provided $q = \sqrt{k_y^2 + k_z^2}$. The integrand in (91) is strongly peaked near the singular point $\zeta = 0$, for which $q^2 + k_x^2 = k^2$. Using standard technique, the formula (91) can be easily further transformed to be applicable in ELC-framework for the charged superluminal particle of 2-nd type propagating in vacuum (i.e. if $\varepsilon = 1$) with a constant speed v_2 higher than that of light ($c_1 \leq v_2 < c_2$):

$$dF = -d\omega \frac{ie^2}{2\pi} \sum \omega \left(\frac{1}{c_2^2} - \frac{1}{v_2^2} \right) \int \frac{d\zeta}{\zeta}, \quad (93)$$

and, respectively, (92) becomes

$$\zeta = q_1^2 - \omega^2 \left(\frac{1}{c_2^2} - \frac{1}{v_2^2} \right), \quad (94)$$

where $q_1 = \sqrt{k_{y1}^2 + k_{z1}^2}$, $q_1^2 + k_{x1}^2 = k_1^2 = \omega^2/c_1^2$, and now $k_{x1}v_2 = \omega$. We have then

$$\zeta = \frac{\omega^2}{c_2^2} \left(\frac{c_2^2}{v_2^2 \cos^2 \theta} - 1 \right) \neq 0, \quad (95)$$

because of $v_2 < c_2$, and that the integral (93) is zero, since the integrand has no poles. Hence, as expected, the *VC-radiation of a charged superluminal particle as it propagates in vacuum is forbidden.*

VI. ACCELERATED MOTION AND LOCAL MS-SUSY

In case of an accelerated ($a = |\vec{a}| \neq 0$) motion of a particle in M_4 , according to (52), we have then

$$\frac{i}{\sqrt{2}} \left(\bar{\theta} \sigma^3 \frac{d^2 \bar{\xi}}{dt^2} - \frac{d^2 \xi}{dt^2} \sigma^3 \bar{\theta} \right) = \frac{d^2 q}{dt^2} = a = \frac{1}{\sqrt{2}} \left(\frac{d^2 \eta^{(+)}}{dt^2} - \frac{d^2 \eta^{(-)}}{dt^2} \right) = \frac{1}{\sqrt{2}} (a^{(+)} - a^{(-)}), \quad a^{(\pm)} = \frac{dv^{(\pm)}}{dt}. \quad (96)$$

So, we may relax the condition $\partial_{\hat{m}} \epsilon = 0$ and promote this symmetry to a local supersymmetry in which the parameter $\epsilon = \epsilon(X^{\hat{m}})$ depends explicitly on $X^{\hat{m}}$. Such a local SUSY can already be read off from the algebra (37) in the form

$$[\epsilon(X)Q, \bar{Q}\bar{\epsilon}(X)] = 2\epsilon(X)\sigma^{\hat{m}}\bar{\epsilon}(X)p_{\hat{m}}, \quad (97)$$

which says that the product of two supersymmetry transformations corresponds to a translation in space-time of which the four momentum $p_{\hat{m}}$ is the generator. Similar to the results of subsection F, the multiplication of two local successive transformations induces the motion

$$g(0, \epsilon(X), \bar{\epsilon}(X))\Omega(X^{\hat{m}}, \Theta, \bar{\Theta}) \longrightarrow (X^{\hat{m}} + i\Theta\sigma^{\hat{m}}\bar{\epsilon}(X) - i\epsilon(X)\sigma^{\hat{m}}\bar{\Theta}, \Theta + \epsilon(X), \bar{\Theta} + \bar{\epsilon}(X)), \quad (98)$$

and, in accord, the transformation (40) is expected to be somewhat of the form

$$[\delta_{\epsilon_1(X)}, \delta_{\epsilon_2(X)}]V \sim \epsilon_1(X)\sigma^{\hat{m}}\bar{\epsilon}_2(X)\partial_{\hat{m}}V, \quad (99)$$

that differ from point to point, namely this is the notion of a general coordinate transformation. Whereupon we see that for the local MS-SUSY to exist it requires the background spaces ($\widetilde{M}_2, \widetilde{M}_4$) to be curved. Thereby, the space \widetilde{M}_4 , in order to become on the same footing with the distorted space \widetilde{M}_2 , refers to the accelerated proper reference frame of a particle, without relation to other matter fields. A useful guide in the construction of local superspace is that it should admit rigid superspace as a limit. The reverse is also expected, since if one starts with a constant parameter ϵ (36) and performs a local Lorentz transformation, then this parameter will in general become space-time dependent as a result of this Lorentz transformation. The mathematical structure of the local MS-SUSY theory has much in common with those used in the geometrical framework of standard supergravity theories. In its simplest version, supergravity was conceived as a quantum field theory whose action included the Einstein-Hilbert term, where the graviton coexists with a fermionic field called gravitino, described by the Rarita-Schwinger kinetic term. The two fields differ in their spin: 2 for the graviton, 3/2 for the gravitino. The different 4D $N = 1$ supergravity multiplets all contain the graviton and the gravitino, but differ by their systems of auxiliary fields. For a detailed discussion we refer to the papers by [7]-[9], [11], [13], [14], [17], [19]-[21]. These fields would transform into each other under local supersymmetry. We may use the usual language which is almost identical to the vierbien formulation of GR with some additional input. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The motion (98) generates the super-general coordinate reparametrization

$$z^M \longrightarrow z'^M = z^M - \zeta^M(z), \quad (100)$$

where $\zeta^M(z)$ are arbitrary functions of z . The dynamical variables of superspace formulation are the frame field $E^A(z)$ and connection Ω . The superspace $(z^M, \Theta, \bar{\Theta})$ has at each point a tangent superspace spanned by the frame field $E^A(z) = dz^M E_M^A(z)$, defined as a 1-form over superspace, with coefficient superfields, generalizing the usual frame, namely supervierbiens $E_M^A(z)$. Here, we use the first half of capital Latin alphabet A, B, \dots to denote the anholonomic indices related to the tangent superspace structure group, which is taken to be just the Lorentz group. The formulation of supergravity in superspace provides a unified description of the vierbein and the Rarita-Schwinger fields. They are identified in a common geometric object, the local frame $E^A(z)$ of superspace. Covariant derivatives with respect to local Lorentz transformations are constructed by means of the spin connection, which is a 1-form in superspace as well. Here we shall forbear to write the details out as the standard theory is so well known. The super-vierbiens E_M^A and spin-connection Ω contain many degrees of freedom. Although some of these are removed by the tangent space and supergeneral coordinate transformations, there still remain many degrees of freedom. There is no general prescription for deducing necessary covariant constraints which if imposed upon the superfields of super-vierbiens and spin-connection will eliminate the component fields. However, some usual constraints can be found using tangent space and supergeneral coordinate transformations of the torsion and curvature covariant tensors, given in appropriate super-gauge. Together with other details of the theory, they can be seen in the textbooks, see e.g. [9], [13]. The final form of transformed super-vierbiens, can be written as

$$E_A^M(z)|_{\Theta=\bar{\Theta}=0} = \begin{pmatrix} e_{\hat{m}}^{\hat{a}}(X) & \frac{1}{2}\psi_{\hat{m}}^{\alpha}(X) & \frac{1}{2}\bar{\psi}_{\hat{m}\dot{\alpha}}(X) \\ 0 & \delta_{\alpha}^{\mu} & 0 \\ 0 & 0 & \delta_{\alpha}^{\mu} \end{pmatrix}, \quad (101)$$

where the fields of graviton $e_{\hat{m}}^{\hat{a}}$ and gravitino $\frac{1}{2}\psi_{\hat{m}}^{\alpha}$, $\frac{1}{2}\bar{\psi}_{\hat{m}\dot{\alpha}}$ cannot be gauged away. Provided, we have

$$e_{\hat{a}}^{\hat{m}} e_{\hat{m}}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}, \quad \psi_{\hat{a}}^{\mu} = e_{\hat{a}}^{\hat{m}} \psi_{\hat{m}}^{\alpha} \delta_{\alpha}^{\mu}, \quad \bar{\psi}_{\hat{a}\dot{\mu}} = e_{\hat{a}}^{\hat{m}} \bar{\psi}_{\hat{m}\dot{\alpha}} \delta_{\dot{\mu}}^{\dot{\alpha}}. \quad (102)$$

The tetrad field $e_{\hat{m}}^{\hat{a}}(X)$ plays the role of a gauge field associated with local transformations. The Majorana type field $\frac{1}{2}\psi_{\hat{m}}^{\alpha}$ is the gauge field related to local supersymmetry. These two fields belong to the same supergravity multiplet which also accommodates auxiliary fields so that the local supersymmetry algebra closes. Under infinitesimal transformations of local supersymmetry, they transformed as

$$\begin{aligned} \delta e_{\hat{m}}^{\hat{a}} &= i(\psi_{\hat{m}} \sigma^{\hat{a}} \zeta - \zeta \sigma^{\hat{a}} \bar{\psi}_{\hat{m}}), \\ \delta \psi_{\hat{m}} &= -2\mathcal{D}_{\hat{m}} \zeta^{\alpha} + i e_{\hat{m}}^{\hat{c}} \left\{ \frac{1}{3} M (\varepsilon \sigma_{\hat{c}} \bar{\zeta})^{\alpha} + b_{\hat{c}} \zeta^{\alpha} + \frac{1}{3} b^{\hat{d}} (\zeta \sigma_{\hat{d}} \bar{\sigma}_{\hat{c}}) \right\}, \end{aligned} \quad (103)$$

etc., where M_4 and $b_{\hat{a}}$ are the auxiliary fields, and $\zeta^{\alpha}(z) = \zeta^{\alpha}(X)$, $\bar{\zeta}^{\alpha}(z) = \bar{\zeta}^{\alpha}(X)$ and $\zeta^{\bar{a}}(z) = 2i[\Theta \sigma^{\hat{a}} \bar{\zeta}(X) - \zeta(X) \sigma^{\hat{a}} \bar{\Theta}]$. The chiral superfields are defined as $\bar{\mathcal{D}}_{\hat{\alpha}} \Phi = 0$, therefore, the components fields are

$$\mathcal{A} = \Phi|_{\Theta=\bar{\Theta}=0}, \quad \psi_{\alpha} = \frac{1}{\sqrt{2}} \mathcal{D}_{\alpha} \Phi|_{\Theta=\bar{\Theta}=0}, \quad \mathcal{F} = -\frac{1}{4} \mathcal{D}^{\alpha} \mathcal{D}_{\alpha} \Phi|_{\Theta=\bar{\Theta}=0}, \quad (104)$$

which carry Lorentz indices. Under infinitesimal transformations of local supersymmetry, they transformed as

$$\begin{aligned} \delta \mathcal{A} &= -\sqrt{2} \zeta^{\alpha} \psi_{\alpha}, \\ \delta \psi_{\alpha} &= -\sqrt{2} \zeta_{\alpha} \mathcal{F} - i\sqrt{2} \sigma_{\alpha\dot{\beta}}^{\hat{a}} \bar{\zeta}^{\dot{\beta}} \hat{\mathcal{D}}_{\hat{a}} \mathcal{A}, \\ \delta \mathcal{F} &= -\frac{1}{3} \sqrt{2} M^* \zeta^{\alpha} \psi_{\alpha} + \bar{\zeta}^{\dot{\alpha}} \left(\frac{1}{6} \sqrt{2} b_{\alpha\dot{\alpha}} \psi^{\dot{\alpha}} - i\sqrt{2} \hat{\mathcal{D}}_{\alpha\dot{\alpha}} \psi^{\dot{\alpha}} \right), \end{aligned} \quad (105)$$

where $\hat{\mathcal{D}}_{\hat{a}}$ is, so-called, super-covariant derivative

$$\begin{aligned} \hat{\mathcal{D}}_{\hat{a}} \mathcal{A} &\equiv e_{\hat{a}}^{\hat{m}} (\partial_{\hat{m}} \mathcal{A} - \frac{i}{\sqrt{2}} \psi_{\hat{m}}^{\mu} \psi_{\mu}), \\ \hat{\mathcal{D}}_{\hat{a}} \psi_{\alpha} &= e_{\hat{a}}^{\hat{m}} (\mathcal{D}_{\hat{m}} \psi_{\alpha} - \frac{1}{\sqrt{2}} \psi_{\hat{m}\alpha} \mathcal{F} - \frac{i}{\sqrt{2}} \bar{\psi}_{\hat{m}}^{\dot{\beta}} \hat{\mathcal{D}}_{\alpha\dot{\beta}} \mathcal{A}). \end{aligned} \quad (106)$$

The graviton and the gravitino form thus the basic multiplet of local MS-SUSY, and one expects the simplest locally supersymmetric model to contain just this multiplet. The spin 3/2 contact term in total Lagrangian arises from equations of motion for the torsion tensor, and that the original Lagrangian itself takes the simpler interpretation of a minimally coupled spin (2, 3/2) theory.

VII. INERTIAL EFFECTS

We would like to place the emphasis on the essential difference arisen between the standard supergravity theories and some rather unusual properties of local MS-SUSY theory. In the framework of the standard supergravity theories,

as in GR, a curvature of the space-time acts on all the matter fields. The source of graviton is the energy-momentum tensor of matter fields, while the source of gravitino is the spin-vector current of supergravity. In the local MS-SUSY theory, unlike the supergravity, a curvature of space-time arises entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a *fictitious gravitation* which can be globally removed by appropriate coordinate transformations. This refers to the particle of interest itself, without relation to other matter fields. The only source of graviton and gravitino, therefore, is the acceleration of a particle, because the MS-SUSY is so constructed as to make these two particles just as being the two bosonic and fermionic states of a particle of interest in the curved background spaces \widetilde{M}_4 and \widetilde{M}_2 , respectively, or vice versa. Whereas, in order to become on the same footing with the distorted space \widetilde{M}_2 , the space \widetilde{M}_4 refers only to the accelerated proper reference frame of a particle. With these physical requirements, a standard Lagrangian consisted of the classical Einstein-Hilbert Lagrangian plus a part which contains the Rarita-Schwinger field and coupling of supergravity with matter superfields evidently no longer holds. Instead we are now looking for an alternative way of implications of local MS-SUSY in the model of accelerated motion and inertial effects. For example, we may with equal justice start from the reverse, which as we mentioned before is also expected. If one starts with a constant parameter ϵ (36) and performs a local Lorentz transformation, which can only be implemented if MS and space-time are curved (deformed/distorted) ($\widetilde{M}_2, \widetilde{M}_4$), then this parameter will in general become space-time dependent as a result of this Lorentz transformation, which readily implies *local* MS-SUSY. In going into practical details of the realistic local MS-SUSY model, it remains to derive the explicit form of the vierbien $e_{\widetilde{m}}^{\hat{a}}(\varrho) \equiv (e_m^a(\varrho), e_{\widetilde{m}}^{\underline{a}}(\varrho))$, which describes *fictitious graviton* as a function of *local rate* $\varrho(\eta, m, f)$ of instantaneously change of the velocity $v^{(\pm)}$ of massive (m) test particle under the unbalanced net force (f). At present, unfortunately, we cannot offer a straightforward recipe for deducing the alluded vierbien $e_{\widetilde{m}}^{\hat{a}}(\varrho)$ in the framework of quantum field theory of MS-supergravity. However, recently it was derived by [4] in the framework of classical physics. Together with other usual aspects of the theory, this illustrates a possible solution to the problems of inertia behind spacetime deformations. Thereby it was argued that a deformation/(distortion of local internal properties) of \widetilde{M}_2 is the origin of inertia effects that can be observed by us. Consequently, the next member of the basic multiplet of local MS-SUSY -*fictitious gravitino*, $\psi_{\widetilde{m}}^\alpha(\varrho)$, will be arisen under infinitesimal transformations of local supersymmetry (103), provided by the local parameters $\zeta^M(a)$ (100).

A. The general space-time deformation/distortion of MS

For the self-contained arguments we first review of certain essential theoretical aspects of a general *distortion of local internal properties of MS* [4], formulated in the framework of classical physics. There was no need for a major revision of the topic but we have taken the opportunity to make one improvement. That is, we show how this recovers the *world-deformation tensor* $\widetilde{\Omega}$, which still has to be put in [5] by hand. To start with, let \underline{V}_2 be 2D semi-Riemann space, which has at each point a tangent space, $\check{T}_{\check{\eta}}\underline{V}_2$, spanned by the anholonomic orthonormal frame field, \check{e} , as a shorthand for the collection of the 2-tuplet $(\check{e}_{(+)}, \check{e}_{(-)})$, where $\check{e}_{\hat{a}} = \check{e}_{\hat{a}}^{\underline{\mu}} \check{e}_{\underline{\mu}}$, with the holonomic frame is given as $\check{e}_{\underline{\mu}} = \check{\partial}_{\underline{\mu}}$. Here, we use the first half of Latin alphabet $\hat{a}, \hat{b}, \hat{c}, \dots = (\pm)$ to denote the anholonomic indices related to the tangent space, and the letters $\underline{\mu}, \underline{\nu} = (\pm)$ to denote the holonomic world indices related either to the space \underline{V}_2 or \widetilde{M}_2 . All magnitudes referred to the space, \underline{V}_2 , will be denoted with an over $' \sim '$. These then define a dual vector, $\check{\vartheta}$, of differential forms, $\check{\vartheta} = \begin{pmatrix} \check{\vartheta}^{(+)} \\ \check{\vartheta}^{(-)} \end{pmatrix}$, as a shorthand for the collection of the $\check{\vartheta}^{\hat{b}} = \check{e}_{\underline{\mu}}^{\hat{b}} \check{\vartheta}^{\underline{\mu}}$, whose values at every point form the dual basis, such that $\check{e}_{\hat{a}} \rfloor \check{\vartheta}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}$, where \rfloor denoting the interior product, namely, this is a C^∞ -bilinear map $\rfloor : \Omega^1 \rightarrow \Omega^0$ with Ω^p denotes the C^∞ -modulo of differential p-forms on \underline{V}_2 . In components $\check{e}_{\hat{a}}^{\underline{\mu}} \check{e}_{\underline{\mu}}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}$. On the manifold, \underline{V}_2 , the tautological tensor field, $i\check{d}$, of type (1,1) can be defined which assigns to each tangent space the identity linear transformation. Thus for any point $\check{\eta} \in \underline{V}_2$, and any vector $\check{\xi} \in \check{T}_{\check{\eta}}\underline{V}_2$, one has $i\check{d}(\check{\xi}) = \check{\xi}$. In terms of the frame field, the $\check{\vartheta}^{\hat{a}}$ give the expression for $i\check{d}$ as $i\check{d} = \check{e}\check{\vartheta} = \check{e}_{(+)} \otimes \check{\vartheta}^{(+)} + \check{e}_{(-)} \otimes \check{\vartheta}^{(-)}$, in the sense that both sides yield $\check{\xi}$ when applied to any tangent vector $\check{\xi}$ in the domain of definition of the frame field. We may consider general transformations of the linear group, $GL(2, R)$, taking any base into any other set of four linearly independent fields. The notation, $\{\check{e}_{\hat{a}}, \check{\vartheta}^{\hat{b}}\}$, will be used below for general linear frames. The holonomic metric can be defined in the semi-Riemann space, \underline{V}_2 , as

$$\check{g} = \check{g}_{\underline{\mu}\underline{\nu}} \check{\vartheta}^{\underline{\mu}} \otimes \check{\vartheta}^{\underline{\nu}} = \check{g}(\check{e}_{\underline{\mu}}, \check{e}_{\underline{\nu}}) \check{\vartheta}^{\underline{\mu}} \otimes \check{\vartheta}^{\underline{\nu}}, \quad (107)$$

with components, $\check{g}_{\underline{\mu}\underline{\nu}} = \check{g}(\check{e}_{\underline{\mu}}, \check{e}_{\underline{\nu}})$ in the dual holonomic base $\{\check{\vartheta}^{\underline{\mu}}\}$. The anholonomic orthonormal frame field, \check{e} , relates \check{g} to the tangent space metric, ${}^*o_{\hat{a}\hat{b}}$, by ${}^*o_{\hat{a}\hat{b}} = \check{g}(\check{e}_{\hat{a}}, \check{e}_{\hat{b}}) = \check{g}_{\underline{\mu}\underline{\nu}} \check{e}_{\hat{a}}^{\underline{\mu}} \check{e}_{\hat{b}}^{\underline{\nu}}$, which has the converse $\check{g}_{\underline{\mu}\underline{\nu}} = {}^*o_{\hat{a}\hat{b}} \check{e}_{\underline{\mu}}^{\hat{a}} \check{e}_{\underline{\nu}}^{\hat{b}}$ because of the relation $\check{e}_{\hat{a}}^{\underline{\mu}} \check{e}_{\underline{\nu}}^{\hat{a}} = \delta_{\underline{\nu}}^{\underline{\mu}}$. A *distortion of local internal properties of MS* comprises then two steps.

1) The linear frame $(\underline{e}_{\underline{m}}; \underline{\vartheta}^{\underline{m}})$ at given point $(p \in \underline{M}_2)$ is undergone the *distortion* transformations conducted by (\check{D}, \check{Y}) and (D, Y) , relating respectively to \underline{V}_2 and $\widetilde{\underline{M}}_2$, which may be recast in the form

$$\check{e}_{\underline{\mu}} = \check{D}_{\underline{\mu}}^{\underline{m}} \bar{e}_{\underline{m}}, \quad \check{\vartheta}^{\underline{\mu}} = \check{Y}_{\underline{m}}^{\underline{\mu}} \bar{\vartheta}^{\underline{m}}, \quad e_{\underline{\mu}} = D_{\underline{\mu}}^{\underline{m}} \bar{e}_{\underline{m}}, \quad \vartheta^{\underline{\mu}} = Y_{\underline{m}}^{\underline{\mu}} \bar{\vartheta}^{\underline{m}}. \quad (108)$$

2) The norm $d\tilde{\eta} \equiv id$ of the infinitesimal displacement $d\tilde{\eta}^{\hat{A}}$ on the general smooth differential 2D-manifold $\widetilde{\mathcal{M}}_2$ can then be written in terms of the space-time structures of V_2 and M_2 as

$$id = e \vartheta = \tilde{\Omega}_{\underline{\mu}}^{\underline{\nu}} \check{e}_{\underline{\nu}} \otimes \check{\vartheta}^{\underline{\mu}} = \Omega_{\hat{b}}^{\hat{a}} \check{e}_{\hat{a}} \otimes \check{\vartheta}^{\hat{b}} = e_{\underline{\mu}} \otimes \vartheta^{\underline{\mu}} = e_{\hat{a}} \otimes \vartheta^{\hat{a}} = \Omega_{\underline{m}}^{\underline{n}} \bar{e}_{\underline{n}} \otimes \bar{\vartheta}^{\underline{m}} \in \widetilde{\underline{M}}_2, \quad (109)$$

where $e = \{e_{\hat{a}} = e_{\hat{a}}^{\underline{\mu}} e_{\underline{\mu}}\}$ is the frame field and $\vartheta = \{\vartheta^{\hat{a}} = e_{\underline{\mu}}^{\hat{a}} \vartheta^{\underline{\mu}}\}$ is the coframe field defined on $\widetilde{\underline{M}}_2$, such that $e_{\hat{a}} \rfloor \vartheta^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}$. Hence the anholonomic deformation tensor $\Omega_{\hat{b}}^{\hat{a}} = \pi_{\hat{c}}^{\hat{a}} \pi_{\hat{b}}^{\hat{c}} = \tilde{\Omega}_{\underline{\mu}}^{\underline{\nu}} \check{e}_{\hat{b}}^{\underline{\mu}} \check{e}_{\hat{c}}^{\underline{\nu}}$ yields local tetrad deformations

$$e_{\hat{c}} = \pi_{\hat{c}}^{\hat{a}} \check{e}_{\hat{a}}, \quad \vartheta^{\hat{c}} = \pi_{\hat{b}}^{\hat{c}} \check{\vartheta}^{\hat{b}}, \quad e \vartheta = e_{\hat{a}} \otimes \vartheta^{\hat{a}} = \Omega_{\hat{b}}^{\hat{a}} \check{e}_{\hat{a}} \otimes \check{\vartheta}^{\hat{b}}. \quad (110)$$

The matrices $\pi(\tilde{\eta}) : = (\pi_{\hat{b}}^{\hat{a}})(\tilde{\eta})$ are referred to as the *first deformation matrices* and the matrices $\gamma_{\hat{c}\hat{d}}(\tilde{\eta}) = {}^*o_{\hat{a}\hat{b}} \pi_{\hat{c}}^{\hat{a}}(\tilde{\eta}) \pi_{\hat{d}}^{\hat{b}}(\tilde{\eta})$, - *second deformation matrices*. The matrices $\pi_{\hat{c}}^{\hat{a}}(\tilde{\eta}) \in GL(2, R) \forall \tilde{\eta}$, in general, give rise to right cosets of the Lorentz group, i.e. they are the elements of the quotient group $GL(2, R)/SO(1, 1)$, because the Lorentz matrices, Λ_s^r , ($r, s = 1, 0$) leave the Minkowski metric invariant. A right-multiplication of $\pi(\tilde{\eta})$ by a Lorentz matrix gives an other deformation matrix. So, all the fundamental geometrical structures on deformed/distorted MS in fact - the metric as much as the coframes and connections - acquire a *deformation/distortion* induced theoretical interpretation. If we deform the tetrad according to (110), in general, we may recast metric as follows:

$$g = {}^*o_{\hat{a}\hat{b}} \pi_{\hat{c}}^{\hat{a}} \pi_{\hat{d}}^{\hat{b}} \check{\vartheta}^{\hat{c}} \otimes \check{\vartheta}^{\hat{d}} = \gamma_{\hat{c}\hat{d}} \check{\vartheta}^{\hat{c}} \otimes \check{\vartheta}^{\hat{d}} = {}^*o_{\hat{a}\hat{b}} \vartheta^{\hat{a}} \otimes \vartheta^{\hat{b}}. \quad (111)$$

The deformed metric can be split as [4]:

$$g_{\underline{\mu}\underline{\nu}}(\pi) = \Upsilon^2(\pi) \check{g}_{\underline{\mu}\underline{\nu}} + \gamma_{\underline{\mu}\underline{\nu}}(\pi), \quad (112)$$

where $\Upsilon(\pi) = \pi_{\hat{a}}^{\hat{a}}$, and

$$\gamma_{\underline{\mu}\underline{\nu}}(\pi) = [\gamma_{\hat{a}\hat{b}} - \Upsilon^2(\pi) {}^*o_{\hat{a}\hat{b}}] \check{e}_{\underline{\mu}}^{\hat{a}} \check{e}_{\underline{\nu}}^{\hat{b}}. \quad (113)$$

B. Model building in the 4D background Minkowski space-time

Here we briefly discuss the RTI in particular case when the relativistic test particle accelerated in the background flat M_4 space under an unbalanced net force other than gravitational, but we refer to the original paper by [4] for more details. To make the remainder of our discussion a bit more concrete, it proves necessary to provide, further, a constitutive ansatz of simple, yet tentative, linear *distortion transformations* of the basis $\underline{e}_{\underline{m}}$ (4) at the point of interest in flat space \underline{M}_2 , which can be written in terms of *local rate* $\varrho(\eta, m, f)$ of instantaneously change of the measure v^A of massive (m) test particle under the unbalanced net force (f) [4]:

$$\begin{aligned} e_{(\tilde{+})}(\varrho) &= D_{(\tilde{+})}^{\underline{m}}(\varrho) \underline{e}_{\underline{m}} = \underline{e}_{(+)} - \varrho(\eta, m, f) v^{(-)} \underline{e}_{(-)}, \\ e_{(\tilde{-})}(\varrho) &= D_{(\tilde{-})}^{\underline{m}}(\varrho) \underline{e}_{\underline{m}} = \underline{e}_{(-)} + \varrho(\eta, m, f) v^{(+)} \underline{e}_{(+)}, \end{aligned} \quad (114)$$

Clearly, these transformations imply a violation of relation (4) ($e_{\underline{\mu}}^2(\varrho) \neq 0$) for the null vectors $\underline{e}_{\underline{m}}$. Whereas we simplify distortion matrices for further use by imposing the constraints

$$D_{\underline{\mu}}^{\underline{m}} = \check{D}_{\underline{\mu}}^{\underline{m}}, \quad \check{Y}_{\underline{m}}^{\underline{\mu}} = \check{D}_{\underline{m}}^{\underline{\mu}}, \quad (115)$$

which yields the partial local tetrad deformations

$$e_{\hat{c}} = \check{e}_{\hat{c}}, \quad \vartheta^{\hat{c}} = \Omega_{\hat{b}}^{\hat{c}} \check{\vartheta}^{\hat{b}}, \quad e \vartheta = e_{\hat{a}} \otimes \vartheta^{\hat{a}} = \Omega_{\hat{b}}^{\hat{a}} \check{e}_{\hat{a}} \otimes \check{\vartheta}^{\hat{b}}. \quad (116)$$

The relation (6) now can be rewritten in terms of space-time variables as

$$id = e \vartheta \equiv d\tilde{q} = \tilde{e}_0 \otimes d\tilde{t} + \tilde{e}_q \otimes d\tilde{q}, \quad (117)$$

where \tilde{e}_0 and \tilde{e}_q are, respectively, the temporal and spatial basis vectors:

$$\tilde{e}_0(\varrho) = \frac{1}{\sqrt{2}} \left[e_{(+)}(\varrho) + e_{(-)}(\varrho) \right], \quad \tilde{e}_q(\varrho) = \frac{1}{\sqrt{2}} \left[e_{(+)}(\varrho) - e_{(-)}(\varrho) \right]. \quad (118)$$

Hence, in the framework of the space-time deformation/distortion theory [4], we can compute the general metric g (111) in \widetilde{M}_2 as

$$g = g_{\tilde{r}\tilde{s}} d\tilde{q}^{\tilde{r}} \otimes d\tilde{q}^{\tilde{s}}, \quad (119)$$

provided

$$g_{\tilde{0}\tilde{0}} = (1 + \frac{\varrho v_q}{\sqrt{2}})^2 - \frac{\varrho^2}{2}, \quad g_{\tilde{1}\tilde{1}} = -(1 - \frac{\varrho v_q}{\sqrt{2}})^2 + \frac{\varrho^2}{2}, \quad g_{\tilde{1}\tilde{0}} = g_{\tilde{0}\tilde{1}} = -\sqrt{2}\varrho. \quad (120)$$

We suppose that a second observer, who makes measurements using a frame of reference $\widetilde{S}_{(2)}$ which is held stationary in curved (deformed/distorted) master space \widetilde{M}_2 , uses for the test particle the corresponding space-time coordinates $\tilde{q}^{\tilde{r}} \left((\tilde{q}^{\tilde{0}}, \tilde{q}^{\tilde{1}}) \equiv (\tilde{t}, \tilde{q}) \right)$. The very concept of the local *absolute acceleration* (in Newton's terminology) is introduced by [4], brought about via the Fermi-Walker transported frames as

$$\vec{a}_{abs} \equiv \vec{e}_q \frac{d(\varrho)}{\sqrt{2}ds_q} = \vec{e}_q \left| \frac{d\varrho}{ds} \right| = \vec{e}_q |\mathbf{a}|. \quad (121)$$

Here we choose the system $S_{(2)}$ in such a way as the axis \vec{e}_q lies along the net 3-acceleration $(\vec{e}_q \parallel \vec{e}_a)$, $(\vec{e}_a = \vec{a}_{net}/|\vec{a}_{net}|)$, \vec{a}_{net} is the local net 3-acceleration of an arbitrary observer with proper linear 3-acceleration \vec{a} and proper 3-angular velocity $\vec{\omega}$ measured in the rest frame: $\vec{a}_{net} = \frac{d\vec{u}}{ds} = \vec{a} \wedge \vec{u} + \vec{\omega} \times \vec{u}$, where \mathbf{u} is the 4-velocity. A magnitude of \vec{a}_{net} can be computed as the simple invariant of the absolute value $|\frac{d\mathbf{u}}{ds}|$ as measured in rest frame:

$$|\mathbf{a}| = \left| \frac{d\mathbf{u}}{ds} \right| = \left(\frac{du^l}{ds}, \frac{du_i}{ds} \right)^{1/2}. \quad (122)$$

Also, following [32, 33], we define an orthonormal frame $e_{\hat{a}}$, carried by an accelerated observer, who moves with proper linear 3-acceleration and $\vec{a}(s)$ and proper 3-rotation $\vec{\omega}(s)$. Particular frame components are $e_{\hat{a}}$, where $\hat{a} = \hat{0}, \hat{1}$, etc. Let the zeroth leg of the frame $e_{\hat{0}}$ be 4-velocity \mathbf{u} of the observer that is tangent to the worldline at a given event $x^l(s)$ and we parameterize the remaining spatial triad frame vectors $e_{\hat{i}}$, orthogonal to $e_{\hat{0}}$, also by (s) . The spatial triad $e_{\hat{i}}$ rotates with proper 3-rotation $\vec{\omega}(s)$. The 4-velocity vector naturally undergoes Fermi-Walker transport along the curve C, which guarantees that $e_{\hat{0}}(s)$ will always be tangent to C determined by $x^l = x^l(s)$:

$$\frac{de_{\hat{a}}}{ds} = -\Phi e_{\hat{a}} \quad (123)$$

where the antisymmetric rotation tensor Φ splits into a Fermi-Walker transport part Φ_{FW} and a spatial rotation part Φ_{SR} :

$$\Phi_{FW}^{lk} = a^l u^k - a^k u^l, \quad \Phi_{SR}^{lk} = u_m \omega_n \varepsilon^{mnlk}. \quad (124)$$

The 4-vector of rotation ω^l is orthogonal to 4-velocity u^l , therefore, in the rest frame it becomes $\omega^l(0, \vec{\omega})$, and ε^{mnlk} is the Levi-Civita tensor with $\varepsilon^{0123} = -1$. So, the resulting metric (119) is reduced to

$$d\tilde{s}_q^2 = \Omega^2(\varrho) ds_q^2, \quad \Omega(\varrho) = 1 + \varrho^2, \quad \varrho^2 = v^2 \varrho'^2, \quad v^2 = v^{(+)} v^{(-)}, \quad \varrho = \sqrt{2} \int_0^{s_q} |\mathbf{a}| ds'_q. \quad (125)$$

Combining (96) and (121), we obtain

$$\varrho = \frac{i}{\gamma_q^2} \left| \left(\varrho \sigma^3 \frac{d\tilde{\xi}}{ds_q} - \frac{d\tilde{\xi}}{ds_q} \sigma^3 \varrho \right) \right|, \quad (126)$$

where $\gamma_q = (1 - v_q^2)^{-1/2}$. The resulting *inertial force* $\vec{f}_{(in)}$ is computed by [4] as

$$\vec{f}_{(in)} = -m \Gamma_{\tilde{r}\tilde{s}}^1(\varrho) \frac{d\tilde{q}^{\tilde{r}}}{ds_q} \frac{d\tilde{q}^{\tilde{s}}}{ds_q} = -\frac{m \vec{a}_{abs}}{\Omega^2(\varrho) \gamma_q}, \quad (127)$$

Whereupon, in case of absence of rotation, the relativistic inertial force reads

$$\vec{f}_{(in)} = -\frac{1}{\Omega^2(\bar{\varrho})\gamma_q\gamma}[\vec{F} + (\gamma - 1)\frac{\vec{v}(\vec{v}\cdot\vec{F})}{|\vec{v}|^2}]. \quad (128)$$

Note that the inertial force arises due to *nonlinear* process of deformation of MS, resulting after all to *linear* relation (128). So, this also ultimately requires that MS should be two dimensional, because in this case we may reconcile the alluded *nonlinear* and *linear* processes by choosing the system $S_{(2)}$ in only allowed way mentioned above. At low velocities $v_q \simeq |\vec{v}| \simeq 0$ and tiny accelerations we usually experience, one has $\Omega(\bar{\varrho}) \simeq 1$, therefore the (128) reduces to the conventional non-relativistic law of inertia

$$\vec{f}_{(in)} = -m\vec{a}_{abs} = -\vec{F}. \quad (129)$$

At high velocities $v_q \simeq |\vec{v}| \simeq 1$ ($\Omega(\bar{\varrho}) \simeq 1$), if $(\vec{v} \cdot \vec{F}) \neq 0$, the inertial force (128) becomes

$$\vec{f}_{(in)} \simeq -\frac{1}{\gamma}\vec{e}_v(\vec{e}_v \cdot \vec{F}), \quad (130)$$

and it vanishes in the limit of the photon ($|\vec{v}| = 1$, $m = 0$). Thus, it takes force to disturb an inertia state, i.e. to make the *absolute acceleration* ($\vec{a}_{abs} \neq 0$). The *absolute acceleration* is due to the real deformation/distortion of the space \underline{M}_2 . The *relative* ($d(\tau_2\varrho)/ds_q = 0$) acceleration (in Newton's terminology) (both magnitude and direction), to the contrary, has nothing to do with the deformation/distortion of the space \underline{M}_2 and, thus, it cannot produce an inertia effects.

C. Beyond the hypothesis of locality

In SR an assumption is required to relate the ideal inertial observers to actual observers that are all noninertial, i.e., accelerated. Therefore, it is a long-established practice in physics to use the hypothesis of locality [33]-[41]), for extension of the Lorentz invariance to accelerated observers in Minkowski space-time. The standard geometrical structures, referred to a noninertial coordinate frame of accelerating and rotating observer in Minkowski space-time, were computed on the base of the assumption that an accelerated observer is pointwise inertial, which in effect replaces an accelerated observer at each instant with a momentarily comoving inertial observer along its worldline. This assumption is known to be an approximation limited to motions with sufficiently low accelerations, which works out because all relevant length scales in feasible experiments are very small in relation to the huge acceleration lengths of the tiny accelerations we usually experience, therefore, the curvature of the worldline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small. However, it seems quite clear that such an approach is a work in progress, which reminds us of a puzzling underlying reality of inertia, and that it will have to be extended to describe physics for arbitrary accelerated observers. Ever since this question has become a major preoccupation of physicists, see e.g. [35]-[44] and references therein. The hypothesis of locality represents strict restrictions, because in other words, it approximately replaces a noninertial frame of reference $\tilde{S}_{(2)}$, which is held stationary in the deformed/distorted space $\tilde{M}_2 \equiv \underline{V}_2^{(\varrho)}$ ($\varrho \neq 0$), with a continuous infinity set of the inertial frames $\{S_{(2)}, S'_{(2)}, S''_{(2)}, \dots\}$ given in the flat \underline{M}_2 ($\varrho = 0$). In this situation the use of the hypothesis of locality is physically unjustifiable. Therefore, it is worthwhile to go beyond the hypothesis of locality with special emphasis on distortion of \underline{M}_2 ($\underline{M}_2 \rightarrow \underline{V}_2^{(\varrho)}$), which we might expect will essentially improve the standard results. The notation will be slightly different from the previous subsection. We now denote the orthonormal frame $e_{\hat{a}}$ (123), carried by an accelerated observer, with the over 'breve' such that

$$\check{e}_{\hat{a}} = \bar{e}_{\hat{a}}^{\mu} \bar{e}_{\mu} = \check{e}_{\hat{a}}^{\mu} \check{e}_{\mu}, \quad \check{\vartheta}^{\hat{b}} = \bar{e}_{\mu}^{\hat{b}} \bar{\vartheta}^{\mu} = \check{e}_{\mu}^{\hat{b}} \check{\vartheta}^{\mu}, \quad (131)$$

with $\bar{e}_{\mu} = \partial_{\mu} = \partial/\partial x^{\mu}$, $\check{e}_{\mu} = \check{\partial}_{\mu} = \partial/\partial \check{x}^{\mu}$, $\bar{\vartheta}^{\mu} = dx^{\mu}$, $\check{\vartheta}^{\mu} = d\check{x}$. Here, following [33, 36], we introduced a geodesic coordinate system \check{x}^{μ} - "coordinates relative to the accelerated observer" (laboratory coordinates), in the neighborhood of the accelerated path. The coframe members $\{\check{\vartheta}^{\hat{b}}\}$ are the objects of dual counterpart: $\check{e}_{\hat{a}} \rfloor \check{\vartheta}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}$. We choose the zeroth leg of the frame, $\check{e}_{\hat{0}}$, as before, to be the unit vector \mathbf{u} that is tangent to the worldline at a given event $x^{\mu}(s)$, where (s) is a proper time measured along the accelerated path by the standard (static inertial) observers in the underlying global inertial frame. The condition of orthonormality for the frame field $\bar{e}_{\hat{a}}^{\mu}$ reads $\eta_{\mu\nu} \bar{e}_{\hat{a}}^{\mu} \bar{e}_{\hat{b}}^{\nu} = o_{\hat{a}\hat{b}} = diag(+ - - -)$. The antisymmetric acceleration tensor Φ_{ab} [36]-[43] is given by

$$\Phi_a{}^b := \bar{e}_{\mu}^{\hat{b}} \frac{d\bar{e}_{\hat{a}}^{\mu}}{ds} = \bar{e}_{\mu}^{\hat{b}} u^{\lambda} \check{\nabla}_{\lambda} \bar{e}_{\hat{a}}^{\mu} = u \rfloor \check{\Gamma}_a{}^b, \quad (132)$$

provided $\check{\Gamma}_a^b = \check{\Gamma}_{a\mu}^b d\check{x}^\mu$, where $\check{\Gamma}_{a\mu}^b$ is the metric compatible, torsion-free Levi-Civita connection. According to (123) and (124), and in analogy with the Faraday tensor, one can identify $\Phi_{ab} \rightarrow (-\mathbf{a}, \omega)$, with $\mathbf{a}(s)$ as the translational acceleration $\Phi_{0i} = -a_i$, and $\omega(s)$ as the frequency of rotation of the local spatial frame with respect to a nonrotating (Fermi-Walker transported) frame $\Phi_{ij} = -\varepsilon_{ijk} \omega^k$. The invariants constructed out of Φ_{ab} establish the acceleration scales and lengths. The hypothesis of locality holds for huge proper acceleration lengths $|I|^{-1/2} \gg 1$ and $|I^*|^{-1/2} \gg 1$, where the scalar invariants are given by $I = (1/2) \Phi_{ab} \Phi^{ab} = -\vec{a}^2 + \vec{\omega}^2$ and $I^* = (1/4) \Phi_{ab}^* \Phi^{ab} = -\vec{a} \cdot \vec{\omega}$ ($\Phi_{ab}^* = \varepsilon_{abcd} \Phi^{cd}$) [36–41]. Suppose the displacement vector $z^\mu(s)$ represents the position of the accelerated observer. According to the hypothesis of locality, at any time (s) along the accelerated worldline the hypersurface orthogonal to the worldline is Euclidean space and we usually describe some event on this hypersurface ("local coordinate system") at x^μ to be at \check{x}^μ , where x^μ and \check{x}^μ are connected via $\check{x}^0 = s$ and

$$x^\mu = z^\mu(s) + \check{x}^i \bar{e}_i^\mu(s). \quad (133)$$

Let $\check{q}^r(\check{q}^0, \check{q}^1)$ be "coordinates relative to the accelerated observer" in the neighborhood of the accelerated path in \underline{M}_2 , with space-time components implying

$$d\check{q}^0 = d\check{x}^0, \quad d\check{q}^1 = |d\check{x}|, \quad \vec{e} = \frac{d\check{x}}{d\check{q}^1} = \frac{d\check{x}}{|d\check{x}|}, \quad \vec{e} \cdot \vec{e} = 1. \quad (134)$$

As long as a locality assumption holds, we may describe, with equal justice, the event at x^μ (133) to be at point \check{q}^r , such that x^μ and \check{q}^r , in full generality, are connected via $\check{q}^0 = s$ and

$$x^\mu = z_q^\mu(s) + \check{q}^1 \bar{\beta}_1^\mu(s), \quad (135)$$

where the displacement vector from the origin reads $dz_q^\mu(s) = \bar{\beta}_0^\mu d\check{q}^0$, and the components $\bar{\beta}_r^\mu$ can be written in terms of \bar{e}_a^μ . Actually, from (133) and (135) we may obtain

$$\begin{aligned} dx^\mu &= dz_q^\mu(s) + d\check{q}^1 \bar{\beta}_1^\mu(s) + \check{q}^1 d\bar{\beta}_1^\mu(s) = \left[\bar{\beta}_0^\mu (1 + \check{q}^1 \check{\varphi}_0) + \bar{\beta}_1^\mu \check{q}^1 \check{\varphi}_1 \right] d\check{q}^0 + \bar{\beta}_1^\mu d\check{q}^1 \\ &\equiv dz^\mu(s) + d\check{x}^i \bar{e}_i^\mu(s) + \check{x}^i d\bar{e}_i^\mu(s) = \left[\bar{e}_0^\mu (1 + \check{x}^i \Phi_i^0) + \bar{e}_j^\mu \check{x}^j \Phi_i^j \right] d\check{x}^0 + \bar{e}_i^\mu d\check{x}^i, \end{aligned} \quad (136)$$

where $d\bar{\beta}_1^\mu(s)$ is written in the basis $\bar{\beta}_a^\mu$ as $d\bar{\beta}_1^\mu = (\check{\varphi}_0 \bar{\beta}_0^\mu + \check{\varphi}_1 \bar{\beta}_1^\mu) d\check{q}^0$. The equation (136) holds by identifying

$$\bar{\beta}_0^\mu (1 + \check{q}^1 \check{\varphi}_0) \equiv \bar{e}_0^\mu (1 + \check{x}^i \Phi_i^0), \quad \bar{\beta}_1^\mu \check{q}^1 \check{\varphi}_1 \equiv \bar{e}_j^\mu \check{x}^j \Phi_i^j, \quad \bar{\beta}_1^\mu d\check{q}^1 \equiv \bar{e}_i^\mu d\check{x}^i. \quad (137)$$

Choosing $\bar{\beta}_0^\mu \equiv \bar{e}_0^\mu$, we have then

$$\check{q}^1 \check{\varphi}_0 = \check{x}^i \Phi_i^0, \quad \bar{\beta}_1^\mu = \bar{e}_i^\mu \check{e}^i, \quad \check{q}^1 \check{\varphi}_1 = \check{x}^i \Phi_i^j \check{e}_j^{-1}, \quad (138)$$

with $\check{e}^j \check{e}_i^{-1} = \delta_i^j$. Consequently, (136) yields the standard metric of semi-Riemannian 4D background space $V_4^{(0)}$ in noninertial system of the accelerating and rotating observer, computed on the base of hypothesis of locality:

$$\begin{aligned} \check{g} &= \eta_{\mu\nu} dx^\mu \otimes dx^\nu = \left[(1 + \vec{a} \cdot \vec{x})^2 + (\vec{\omega} \cdot \vec{x})^2 - (\vec{\omega} \cdot \vec{\omega})(\vec{x} \cdot \vec{x}) \right] d\check{x}^0 \otimes d\check{x}^0 - \\ &2 (\vec{\omega} \wedge \vec{x}) \cdot d\vec{x} \otimes d\check{x}^0 - d\vec{x} \otimes d\vec{x}, \end{aligned} \quad (139)$$

This metric was derived by [34] and [35], in agreement with [45]–[47] (see also [36–41]). We see that the hypothesis of locality leads to the 2D semi-Riemannian space, $\underline{V}_2^{(0)}$, with the incomplete metric \check{g} ($\varrho = 0$):

$$\check{g} = \left[(1 + \check{q}^1 \check{\varphi}_0)^2 - (\check{q}^1 \check{\varphi}_1)^2 \right] d\check{q}^0 \otimes d\check{q}^0 - 2 (\check{q}^1 \check{\varphi}_1) d\check{q}^1 \otimes d\check{q}^0 - d\check{q}^1 \otimes d\check{q}^1, \quad (140)$$

Therefore, our strategy now is to deform the metric (140) by carrying out an additional deformation of semi-Riemannian 4D background space $V_4^{(0)} \rightarrow \tilde{M}_4 \equiv V_4^{(\varrho)}$, in order it becomes on the same footing with the complete metric g ($\varrho \neq 0$) (119) of the distorted space $\tilde{\underline{M}}_2 \equiv \underline{V}_2^{(\varrho)}$. Let the Latin letters $\hat{r}, \hat{s}, \dots = 0, 1$ be the anholonomic indices referred to the anholonomic frame $e_{\hat{r}} = e_{\hat{r}}^s \partial_{\hat{s}}$, defined on the $\underline{V}_2^{(\varrho)}$, with $\partial_{\hat{s}} = \partial / \partial \tilde{q}^{\hat{s}}$ as the vectors tangent to the coordinate lines. So, a smooth differential 2D-manifold $\underline{V}_2^{(\varrho)}$ has at each point $\tilde{q}^{\hat{s}}$ a tangent space $\tilde{T}_{\tilde{q}} \underline{V}_2^{(\varrho)}$, spanned by the frame, $\{e_{\hat{r}}\}$, and the coframe members $\vartheta^{\hat{r}} = e_{\hat{s}}^{\hat{r}} d\tilde{q}^{\hat{s}}$, which constitute a basis of the covector space $\tilde{T}_{\tilde{q}}^* \underline{V}_2^{(\varrho)}$. All

this nomenclature can be given for $\underline{V}_2^{(0)}$ too. Then, we may compute corresponding vierbein fields $\check{e}_r^{\hat{s}}$ and $e_r^{\hat{s}}$ from the equations

$$\check{g}_{rs} = \check{e}_r^{\hat{r}'} \check{e}_s^{\hat{s}'} o_{\hat{r}'\hat{s}'}, \quad g_{\bar{r}\bar{s}}(\varrho) = e_r^{\hat{r}'}(\varrho) e_s^{\hat{s}'}(\varrho) o_{\hat{r}'\hat{s}'}, \quad (141)$$

with \check{g}_{rs} (140) and $g_{\bar{r}\bar{s}}(\varrho)$ (120). Hence

$$\begin{aligned} \check{e}_0^{\hat{0}} &= 1 + \vec{a} \cdot \vec{x}, & \check{e}_0^{\hat{1}} &= \vec{\omega} \wedge \vec{x}, & \check{e}_1^{\hat{0}} &= 0, & \check{e}_1^{\hat{1}} &= 1, \\ e_0^{\hat{0}}(\varrho) &= 1 + \frac{\varrho v_q}{\sqrt{2}}, & e_0^{\hat{1}}(\varrho) &= \frac{\varrho}{\sqrt{2}}, & e_1^{\hat{0}}(\varrho) &= -\frac{\varrho}{\sqrt{2}}, & e_1^{\hat{1}}(\varrho) &= 1 - \frac{\varrho v_q}{\sqrt{2}}. \end{aligned} \quad (142)$$

Since a distortion $\underline{M}_2 \rightarrow \widetilde{\underline{M}}_2$ may affect only the \underline{M}_2 -part of the components $\bar{\beta}_{\hat{r}}^{\mu}$, without relation to the 4D background space-time part, therefore, a deformation $V_4^{(0)} \rightarrow V_4^{(\varrho)}$ is equivalent to a straightforward generalization $\bar{\beta}_{\hat{r}}^{\mu} \rightarrow \beta_{\hat{r}}^{\mu}(\varrho)$, where

$$\beta_{\hat{r}}^{\mu}(\varrho) = E_{\hat{r}}^{\hat{s}}(\varrho) \bar{\beta}_{\hat{s}}^{\mu}, \quad E_{\hat{r}}^{\hat{s}}(\varrho) : = e_{\hat{r}'}^{\hat{s}}(\varrho) \check{e}_{\hat{r}'}^{\hat{s}}. \quad (143)$$

Consequently, the (143) gives a generalization of (133) as

$$x^{\mu} \rightarrow x_{(\varrho)}^{\mu} = z_{(\varrho)}^{\mu}(s) + \check{x}^i e_{\hat{i}}^{\mu}(s), \quad (144)$$

provided, as before, \check{x}^{μ} denotes the coordinates relative to the accelerated observer in 4D background space $V_4^{(\varrho)}$, and according to (137), we have

$$e_{\hat{0}}^{\mu}(\varrho) = \beta_{\hat{0}}^{\mu}(\varrho), \quad e_{\hat{i}}^{\mu}(\varrho) = \beta_{\hat{i}}^{\mu}(\varrho) \check{e}_i^{-1}. \quad (145)$$

A displacement vector from the origin is then $dz_{\varrho}^{\mu}(s) = e_{\hat{0}}^{\mu}(\varrho) d\check{x}^0$, Combining (143) and (145), and inverting $e_r^{\hat{s}}(\varrho)$ (142), we obtain $e_{\hat{a}}^{\mu}(\varrho) = \pi_{\hat{a}}^{\hat{b}}(\varrho) \bar{e}_{\hat{b}}^{\mu}$, where

$$\begin{aligned} \pi_{\hat{0}}^{\hat{0}}(\varrho) &\equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1} (1 - \frac{\varrho v_q}{\sqrt{2}}) (1 + \vec{a} \cdot \vec{x}), & \pi_{\hat{0}}^{\hat{i}}(\varrho) &\equiv -(1 + \frac{\varrho^2}{2\gamma_q^2})^{-1} \frac{\varrho}{\sqrt{2}} \check{e}^i (1 + \vec{a} \cdot \vec{x}), \\ \pi_{\hat{i}}^{\hat{0}}(\varrho) &\equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1} \left[(\vec{\omega} \wedge \vec{x}) (1 - \frac{\varrho v_q}{\sqrt{2}}) - \frac{\varrho}{\sqrt{2}} \right] \check{e}_i^{-1}, & \pi_{\hat{i}}^{\hat{j}}(\varrho) &= \delta_i^j \pi(\varrho), \\ \pi(\varrho) &\equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1} \left[(\vec{\omega} \wedge \vec{x}) \frac{\varrho}{\sqrt{2}} + 1 + \frac{\varrho v_q}{\sqrt{2}} \right]. \end{aligned} \quad (146)$$

Thus,

$$dx_{\varrho}^{\mu} = dz_{\varrho}^{\mu}(s) + d\check{x}^i e_{\hat{i}}^{\mu} + \check{x}^i de_{\hat{i}}^{\mu}(s) = (\tau^{\hat{b}} d\check{x}^0 + \pi_{\hat{i}}^{\hat{b}} d\check{x}^i) \bar{e}_{\hat{b}}^{\mu}, \quad (147)$$

where

$$\tau^{\hat{b}} \equiv \pi_{\hat{0}}^{\hat{b}} + \check{x}^i \left(\pi_{\hat{i}}^{\hat{a}} \Phi_a^{\hat{b}} + \frac{d\pi_{\hat{i}}^{\hat{b}}}{ds} \right). \quad (148)$$

Hence, in general, the metric in noninertial frame of arbitrary accelerating and rotating observer in Minkowski space-time is

$$g(\varrho) = \eta_{\mu\nu} dx_{\varrho}^{\mu} \otimes dx_{\varrho}^{\nu} = W_{\mu\nu}(\varrho) d\check{x}^{\mu} \otimes d\check{x}^{\nu}, \quad (149)$$

which can be conveniently decomposed according to

$$\begin{aligned} W_{00}(\varrho) &= \pi^2 \left[(1 + \vec{a} \cdot \vec{x})^2 + (\vec{\omega} \cdot \vec{x})^2 - (\vec{\omega} \cdot \vec{\omega})(\vec{x} \cdot \vec{x}) \right] + \gamma_{00}(\varrho), \\ W_{0i}(\varrho) &= -\pi^2 (\vec{\omega} \wedge \vec{x})^i + \gamma_{0i}(\varrho), \quad W_{ij}(\varrho) = -\pi^2 \delta_{ij} + \gamma_{ij}(\varrho), \end{aligned} \quad (150)$$

and that

$$\begin{aligned} \gamma_{00}(\varrho) &= \pi \left[(1 + \vec{a} \cdot \vec{x}) \zeta^0 - (\vec{\omega} \wedge \vec{x}) \cdot \vec{\zeta} \right] + (\zeta^0)^2 - (\vec{\zeta})^2, & \gamma_{0i}(\varrho) &= -\pi \zeta^i + \tau^{\hat{0}} \pi_{\hat{i}}^{\hat{0}}, \\ \gamma_{ij}(\varrho) &= \pi_{\hat{i}}^{\hat{0}} \pi_{\hat{j}}^{\hat{0}}, & \zeta^0 &= \pi \left(\tau^{\hat{0}} - 1 - \vec{a} \cdot \vec{x} \right), & \vec{\zeta} &= \pi \left(\vec{\tau} - \vec{\omega} \wedge \vec{x} \right). \end{aligned} \quad (151)$$

As we expected, according to (149)- (151), the metric $g(\varrho)$ is decomposed in the form of (112):

$$g(\varrho) = \pi^2(\varrho) \check{g} + \gamma(\varrho), \quad (152)$$

where $\gamma(\varrho) = \gamma_{\mu\nu}(\varrho) d\check{x}^\mu \otimes d\check{x}^\nu$ and $\Upsilon(\varrho) = \pi_{\check{a}}^{\check{a}}(\varrho) = \pi(\varrho)$. In general, the geodesic coordinates are admissible as long as

$$\left(1 + \vec{a} \cdot \vec{x} + \frac{\zeta^0}{\pi}\right)^2 > \left(\vec{\omega} \wedge \vec{x} + \frac{\vec{\zeta}}{\pi}\right)^2. \quad (153)$$

The equations (139) and (149) say that the vierbein fields, with entries $\eta_{\mu\nu} \bar{e}_{\check{a}}^\mu \bar{e}_{\check{b}}^\nu = o_{\check{a}\check{b}}$ and $\eta_{\mu\nu} e_{\check{a}}^\mu e_{\check{b}}^\nu = \gamma_{\check{a}\check{b}}$ lead to the relations

$$\check{g} = o_{\check{a}\check{b}} \check{\vartheta}^{\check{a}} \otimes \check{\vartheta}^{\check{b}}, \quad g = o_{\check{a}\check{b}} \vartheta^{\check{a}} \otimes \vartheta^{\check{b}} = \gamma_{\check{a}\check{b}} \check{\vartheta}^{\check{a}} \otimes \check{\vartheta}^{\check{b}} = (\Omega_{\check{a}}^{\check{c}} \Omega_{\check{b}}^{\check{d}} o_{\check{c}\check{d}}) \bar{\vartheta}^{\check{a}} \otimes \bar{\vartheta}^{\check{b}}, \quad (154)$$

and that (136) and (147) readily give the coframe fields:

$$\begin{aligned} \check{\vartheta}^{\check{b}} &= \bar{e}_{\mu}^{\check{b}} dx^\mu = \check{e}_{\mu}^{\check{b}} d\check{x}^\mu, \quad \check{e}_{\check{0}}^{\check{b}} = N_{\check{0}}^{\check{b}}, \quad \check{e}_{\check{i}}^{\check{b}} = N_{\check{i}}^{\check{b}}, \\ \vartheta^{\check{b}} &= \bar{e}_{\mu}^{\check{b}} dx_{\check{e}}^\mu = e_{\mu}^{\check{b}} d\check{x}^\mu = \pi_{\check{a}}^{\check{b}} \check{\vartheta}^{\check{a}}, \quad e_{\check{0}}^{\check{b}} = \tau^{\check{b}}, \quad e_{\check{i}}^{\check{b}} = \pi_{\check{i}}^{\check{b}}. \end{aligned} \quad (155)$$

where $N_{\check{0}}^0 = N \equiv \left(1 + \vec{a} \cdot \vec{x}\right)$, $N_{\check{i}}^0 = 0$, $N_{\check{0}}^{\check{i}} = N^{\check{i}} \equiv \left(\vec{\omega} \cdot \vec{x}\right)^{\check{i}}$, $N_{\check{i}}^{\check{j}} = \delta_{\check{i}}^{\check{j}}$. In the standard (3+1)-decomposition of space-time, N and $N^{\check{i}}$ are known as *lapse function* and *shift vector*, respectively [48]. Hence, we may easily recover the frame field $e_{\check{a}} = e_{\check{a}}^\mu \check{e}_\mu = \pi_{\check{a}}^{\check{b}} \check{e}_{\check{b}}$ by inverting (155):

$$\begin{aligned} e_{\check{0}}(\varrho) &= \frac{\pi(\varrho)}{\pi(\varrho) \tau^0(\varrho) - \pi_{\check{k}}^0(\varrho) \tau^{\check{k}}(\varrho)} \check{e}_0 - \frac{\tau^{\check{i}}(\varrho)}{\pi(\varrho) \tau^0(\varrho) - \pi_{\check{k}}^0(\varrho) \tau^{\check{k}}(\varrho)} \check{e}_{\check{i}}, \\ e_{\check{i}}(\varrho) &= -\frac{\pi_{\check{i}}^0(\varrho)}{\pi(\varrho) \tau^0(\varrho) - \pi_{\check{k}}^0(\varrho) \tau^{\check{k}}(\varrho)} \check{e}_0 + \pi^{-1}(\varrho) \left[\delta_{\check{i}}^{\check{j}} + \frac{\tau^{\check{j}}(\varrho) \pi_{\check{i}}^0(\varrho)}{\pi(\varrho) \tau^0(\varrho) - \pi_{\check{k}}^0(\varrho) \tau^{\check{k}}(\varrho)} \right] \check{e}_{\check{j}}. \end{aligned} \quad (156)$$

A *generalized transport* for deformed frame $e_{\check{a}}$, which includes both the Fermi-Walker transport and distortion of \underline{M}_2 , can be written in the form

$$\frac{de_{\check{a}}^\mu}{ds} = \tilde{\Phi}_{\check{a}}^{\check{b}} e_{\check{b}}^\mu, \quad (157)$$

where a *deformed acceleration tensor* $\tilde{\Phi}_{\check{a}}^{\check{b}}$ concisely is given by

$$\tilde{\Phi} = \frac{d \ln \pi}{ds} + \pi \Phi \pi^{-1}. \quad (158)$$

Thus, we derive the tetrad fields $e_{\check{r}}^{\check{s}}(\varrho)$ (142) and $e_{\check{a}}^\mu(\varrho)$ (156) as a function of *local rate* ϱ of instantaneously change of a constant velocity (both magnitude and direction) of a massive particle in M_4 under the unbalanced net force, describing corresponding *fictitious graviton*. Then, the *fictitious gravitino*, $\psi_{\check{m}}^\alpha(\varrho)$, will be arisen under infinitesimal transformations of local supersymmetry (103), provided by the local parameters $\zeta^M(a)$ (100).

D. Involving the background semi-Riemann space V_4 ; Justification for the introduction of the WPE

We can always choose *natural coordinates* $X^\alpha(T, X, Y, Z) = (T, \vec{X})$ with respect to the axes of the local free-fall coordinate frame $S_4^{(l)}$ in an immediate neighbourhood of any space-time point $(\check{x}_p) \in V_4$ in question of the background semi- Riemann space, V_4 , over a differential region taken small enough so that we can neglect the spatial and temporal variations of gravity for the range involved. The values of the metric tensor $\check{g}_{\mu\nu}$ and the affine connection $\check{\Gamma}_{\mu\nu}^\lambda$ at the point (\check{x}_p) are necessarily sufficient information for determination of the natural coordinates $X^\alpha(\check{x}^\mu)$ in the small region of the neighbourhood of the selected point [49]. Then the whole scheme outlined in the previous subsections (a) and (b) will be held in the frame $S_4^{(l)}$. The general *inertial force* computed by [4] reads

$$\check{f}_{(in)}^\alpha = -\frac{m \check{a}_{gbs}}{\Omega^2(\check{\varrho}) \gamma_q} = -\frac{\check{e}_f}{\Omega^2(\check{\varrho}) \gamma_q} |f_{(l)}^\alpha - m \frac{\partial X^\alpha}{\partial \check{x}^\sigma} \check{\Gamma}_{\mu\nu}^\sigma \frac{d\check{x}^\mu}{dS} \frac{d\check{x}^\nu}{dS}|. \quad (159)$$

Whereas, as before, the two systems S_2 and $S_4^{(l)}$ can be chosen in such a way as the axis \vec{e}_q of $S_{(2)}$ lies ($\vec{e}_q = \vec{e}_f$) along the acting net force $\vec{f} = \vec{f}_{(l)} + \vec{f}_{g(l)}$, while the time coordinates in the two systems are taken the same, $q^0 = t = X^0 = T$.

Here $\vec{f}_{(l)}$ is the SR value of the unbalanced relativistic force other than gravitational and $\vec{f}_{g(l)}$ is the gravitational force given in the frame $S_4^{(l)}$. Despite of totally different and independent sources of gravitation and inertia, at $f_{(l)}^\alpha = 0$, the (159) establishes the independence of free-fall ($v_q = 0$) trajectories of the mass, internal composition and structure of bodies. This furnishes a justification for the introduction of the WPE. A remarkable feature is that, although the inertial force has a nature different than the gravitational force, nevertheless both are due to a distortion of the local inertial properties of, respectively, 2D \underline{M}_2 and 4D-background space.

E. The inertial effects in the background post Riemannian geometry

If the nonmetricity tensor $N_{\lambda\mu\nu} = -\mathcal{D}_\lambda g_{\mu\nu} \equiv -g_{\mu\nu;\lambda}$ does not vanish, the general formula for the affine connection written in the space-time components is [50]

$$\Gamma_{\mu\nu}^\rho = \overset{\circ}{\Gamma}_{\mu\nu}^\rho + K_{\mu\nu}^\rho - N_{\mu\nu}^\rho + \frac{1}{2} N_{(\mu}{}^\rho{}_{\nu)}, \quad (160)$$

where the metric alone determines the torsion-free Levi-Civita connection $\overset{\circ}{\Gamma}_{\mu\nu}^\rho$, $K_{\mu\nu}^\rho := 2Q_{(\mu\nu)}^\rho + Q_{\mu\nu}^\rho$ is the non-Riemann part - the affine *contortion tensor*. The torsion, $Q_{\mu\nu}^\rho = \frac{1}{2} T_{\mu\nu}^\rho = \Gamma_{[\mu\nu]}^\rho$ given with respect to a holonomic frame, $d\vartheta^\rho = 0$, is a third-rank tensor, antisymmetric in the first two indices, with 24 independent components. We now compute the relativistic inertial force for the motion of the matter, which is distributed over a small region in the U_4 space and consists of points with the coordinates x^μ , forming an extended body whose motion in the space, U_4 , is represented by a world tube in space-time. Suppose the motion of the body as a whole is represented by an arbitrary timelike world line γ inside the world tube, which consists of points with the coordinates $\tilde{X}^\mu(\tau)$, where τ is the proper time on γ . Define

$$\delta x^\mu = x^\mu - \tilde{X}^\mu, \quad \delta x^0 = 0, \quad u^\mu = \frac{d\tilde{X}^\mu}{d\tau}. \quad (161)$$

The *Papapetrou equation of motion for the modified momentum* ([50]-[53]) is

$$\frac{\overset{\circ}{D}\Theta^\nu}{\mathcal{D}s} = -\frac{1}{2} \overset{\circ}{R}{}^\nu{}_{\mu\sigma\rho} u^\mu J^{\sigma\rho} - \frac{1}{2} N_{\mu\rho\lambda} K^{\mu\rho\lambda;\nu}, \quad (162)$$

where $K_{\nu\lambda}^\mu$ is the contortion tensor,

$$\Theta^\nu = P^\nu + \frac{1}{u^0} \overset{\circ}{\Gamma}{}^\nu{}_{\mu\rho} (u^\mu J^{\rho 0} + N^{0\mu\rho}) - \frac{1}{2u^0} K_{\mu\rho}{}^\nu N^{\mu\rho 0} \quad (163)$$

is referred to as the *modified 4-momentum*, $P^\lambda = \int \tau^{\lambda 0} d\Omega$ is the ordinary 4-momentum, $d\Omega := dx^4$, and the following integrals are defined:

$$\begin{aligned} M^{\mu\rho} &= u^0 \int \tau^{\mu\rho} d\Omega, \quad M^{\mu\nu\rho} = -u^0 \int \delta x^\mu \tau^{\nu\rho} d\Omega, \quad N^{\mu\nu\rho} = u^0 \int s^{\mu\nu\rho} d\Omega, \\ J^{\mu\rho} &= \int (\delta x^\mu \tau^{\rho 0} - \delta x^\rho \tau^{\mu 0} + s^{\mu\rho 0}) d\Omega = \frac{1}{u^0} (-M^{\mu\rho 0} + M^{\rho\mu 0} + N^{\mu\rho 0}), \end{aligned} \quad (164)$$

where $\tau^{\mu\rho}$ is the energy-momentum tensor for particles, $s^{\mu\nu\rho}$ is the spin density. The quantity $J^{\mu\rho}$ is equal to $\int (\delta x^\mu \tau^{kl} - \delta x^\rho \tau^{\mu\lambda} + s^{\mu\rho\lambda}) dS_\lambda$ taken for the volume hypersurface, so it is a tensor, which is called the *total spin tensor*. The quantity $N^{\mu\nu\rho}$ is also a tensor. The relation $\delta x^0 = 0$ gives $M^{0\nu\rho} = 0$. It was assumed that the dimensions of the body are small, so integrals with two or more factors δx^μ multiplying $\tau^{\nu\rho}$ and integrals with one or more factors δx^μ multiplying $s^{\nu\rho\lambda}$ can be neglected. The *Papapetrou equations of motion for the spin* ([50]-[53]) is

$$\frac{\overset{\circ}{D}}{\mathcal{D}s} J^{\lambda\nu} = u^\nu \Theta^\lambda - u^\lambda \Theta^\nu + K_{\mu\rho}^\lambda N^{\nu\mu\rho} + \frac{1}{2} K_{\mu\rho}{}^\lambda N^{\mu\nu\rho} - K_{\mu\rho}^\nu N^{\lambda\mu\rho} - \frac{1}{2} K_{\mu\rho}{}^\nu N^{\mu\rho\lambda}. \quad (165)$$

Computing from (162), in general, the relativistic inertial force, exerted on the extended spinning body moving in the RC space U_4 , can be found to be

$$\begin{aligned} \vec{f}_{(in)}(x) &= -\frac{m\vec{a}_{abs}(x)}{\Omega^2(\bar{\vartheta})\gamma_q} = -m \frac{\vec{e}_f}{\Omega^2(\bar{\vartheta})\gamma_q} \left[\frac{1}{m} f_{(l)}^\alpha - \frac{\partial X^\alpha}{\partial x^\mu} \left[\overset{\circ}{\Gamma}{}^\mu{}_{\nu\lambda} u^\nu u^\lambda + \right. \right. \\ &\quad \left. \left. \frac{1}{u^0} \overset{\circ}{\Gamma}{}^\mu{}_{\nu\rho} (u^\nu J^{\rho 0} + N^{0\nu\rho}) - \frac{1}{2u^0} K_{\nu\rho}{}^\mu N^{\nu\rho 0} + \frac{1}{2} \overset{\circ}{R}{}^\mu{}_{\nu\sigma\rho} u^\nu J^{\sigma\rho} + \frac{1}{2} N_{\nu\rho\lambda} K^{\nu\rho\lambda;\mu} \right] \right]. \end{aligned} \quad (166)$$

VIII. CONCLUDING REMARKS

We present a standard Lorentz code of motion in a new perspective of supersymmetry. In this, we explore the intermediate, so-called, *motion* state for a particle moving through the two infinitesimally closed points of original space. The Schwinger transformation function for these points is understood as the successive processes of annihilation of a particle at initial point and time, i.e. the transition from the initial state to the intermediate *motion* state, and the creation of a particle at final point and time, i.e. the subsequent transition from the intermediate *motion* state to the final state. The latter is defined on the *master space*, $MS \equiv \underline{M}_2$, which is prescribed to each particle, without relation to every other particle. Exploring the rigid double transformations of MS-SUSY, we derive SLC as the individual code of a particle in terms of spinors referred to MS. This allows to introduce the physical finite *time interval* between two events, as integer number of the *duration time* of atomic double transition of a particle from M_4 to \underline{M}_2 and back. The theories with extended $N_{max} = 4$ supersymmetries, as renormalizable flat-space field theories, if only such symmetries are fundamental to nature, lead to the model of ELC in case of the apparent violations of SLC, the possible manifestations of which arise in a similar way in all particle sectors. We show that in the ELC-framework the propagation of the superluminal particle could be consistent with causality, and give a justification of forbiddance of Vavilov-Cherenkov radiation/or analog processes in vacuum. In the framework of local MS-SUSY, we address the inertial effects. The local MS-SUSY can only be implemented if \widetilde{M}_2 and \widetilde{M}_4 are curved (deformed). Whereas the space \widetilde{M}_4 , in order to become on the same footing with the distorted space \widetilde{M}_2 , refers to the accelerated reference frame of a particle, without relation to other matter fields. So, unlike gravitation, a curvature of space-time arises entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a *fictitious gravitation* which can be globally removed by appropriate coordinate transformations. The only source of graviton and gravitino, therefore, is the acceleration of a particle, because the MS-SUSY is so constructed as to make these two particles just as being the two bosonic and fermionic states of a particle of interest in the background spaces M_4 and \underline{M}_2 , respectively, or vice versa. Therefore, a coupling of supergravity with matter superfields evidently is absent in resulting theory. Instead, we argue that a deformation/(distortion of local internal properties) of MS is the origin of inertia effects that can be observed by us. In the framework of classical physics we briefly discuss the model of inertia effects and go beyond the hypothesis of locality. This allows to improve essentially the relevant geometrical structures referred to the noninertial frame in Minkowski space-time for an arbitrary velocities and characteristic acceleration lengths. Despite the totally different and independent physical sources of gravitation and inertia, this approach furnishes justification for the introduction of the WPE. Consequently, we relate the inertia effects to the more general post-Riemannian geometry.

-
- [1] S. Drake, Galileo at work, Chicago, University of Chicago Press (1978), (<http://en.wikipedia.org/wiki/Galileo>).
 - [2] I. Newton, Philosophiae Naturalis Principia Mathematica, (1687), (<http://plato.stanford.edu/entries/newton-principia>).
 - [3] J. Norton, What was Einsteins principle of equivalence?, *Stud. Hist. Phil. Sci.* **16** 203 (1985).
 - [4] G. Ter-Kazarian, Spacetime deformation induced inertia effects, *Advances in Mathematical Physics*, Vol. 2012, Article ID 692030, 41 pages, doi:10.1155/2012/692030, Hindawi Publ. Corporation (2012).
 - [5] G. Ter-Kazarian, Two-step spacetime deformation induced dynamical torsion, *Class. Quantum Grav.*, **28**, 055003 (19pp.), (2011); arXiv:1102.2491[gr-qc].
 - [6] H.K. Dreiner, H.E. Haber and S.P. Martin, Supersymmetry, CUP draft Sept. pp.1-272 (2004).
 - [7] P. Fayet and S. Ferrara, Supersymmetry, *Physics Reports*, **32** pp.249-334 (1977).
 - [8] P. van Nieuwenhuizen, Supergravity, *Physics Reports*, **68** pp.189-398 (1981).
 - [9] J. Wess and J. Bagger, Supersymmetry and Supergravity, Princeton University Press, Princeton, New Jersey, pp.1-181 (1983).
 - [10] M S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, Superspace: Or One Thousand and One Lessons in Supersymmetry, Benjamin/Cumming, London, pp.1-548 (1983).
 - [11] H.P. Nilles, Supersymmetry, Supergravity and Particle Physics, *Physics Reports*, **110** pp.1-162 (1984).
 - [12] M.F. Sohnius, Introducing Supersymmetry, *Physics Reports*, **128** pp.39-204 (1985).
 - [13] P. West, Introduction to Supersymmetry and Supergravity, World Scientific, Singapore, pp.1-289 (1987).
 - [14] M. Jacob, Supersymmetry and Supergravity, North Holland, Amsterdam, (1987).
 - [15] H. Baer and X. Tata, Weak Scale Supersymmetry: From Superfields to Scattering Events, (Cambridge: Cambridge University Press) (2006).
 - [16] I.J.R. Aitchison, Supersymmetry in particle physics: an elementary introduction, SLAC Report SLAC-R-865 (2007).
 - [17] P. Binetruy, G. Girardi, R. Rimm, Supergravity couplings: a geometric formulation, *Physics Reports*, **343** pp.255-462 (2001).

- [18] M. Drees, Supersymmetry, (Oxford: Oxford University Press) (2007).
- [19] M. Dine, Supersymmetry and String Theory: Beyond the Standard Model, (Cambridge: Cambridge University Press) (2007).
- [20] P. van Nieuwenhuizen, D.Z. Freedman and S. Ferrara, Progress toward a Theory of Supergravity, Phys. Rev., **D13** 3214-3218 (1976).
- [21] S. Deser and B. Zumino, Consistent Supergravity, Phys. Lett. **B62** 335 (1976).
- [22] R.N. Mohapatra, Unification and Supersymmetry, The Frontiers of Quark-Lepton Physics, 3th ed., Springer-Verlag New York, Inc., pp.1-421 (2002).
- [23] M. Dine and J.D. Mason, Supersymmetry and its dynamical breaking, Rep. Prog. Phys., **74**, pp.1-29 056201 (2011).
- [24] J. Wess and B. Zumino, A Lagrangian model invariant under supergauge transformations, Phys. Lett. **49B** 52 (1974).
- [25] S. Weinberg, Nonlinear Realizations of Chiral Symmetry, Phys. Rev., **166** 1568 (1968).
- [26] S. Coleman, J. Wess and B. Zumino, Structure of phenomenological Lagrangians.I, Phys. Rev. **177** 2239 (1969).
- [27] C. Callan, S. Coleman, J. Wess and B. Zumino, Structure of phenomenological Lagrangians. 2, Phys. Rev. **177** 2247 (1969).
- [28] D.V. Volkov, Phenomenological Lagrangians, Sov. J. Particles and Nuclei **4** 1-17 (1973).
- [29] V.I. Ogievetsky, in Proc. of X-th Winter School of Theoretical Physics in Karpacz, Wroclaw (1974).
- [30] A. Salam and J. Strathdee, Supergauge Transformations, Nucl. Phys. **B76** 477 (1974).
- [31] L.D.Landau and E.M. Lifshitz, Electrodynamics of Continuous Media, Theoretical physics, vol.VIII. Moscow, Nauka (1992).
- [32] J.L. Synge, Relativity: The General Theory, North-Holland, Amsterdam (1960).
- [33] C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation (Freeman, San Francisco) (1973).
- [34] F.W. Hehl, W.-T. Ni, Inertial effects of a Dirac particle, Phys. Rev. D, **42** pp.2045-2048 (1990).
- [35] F.W. Hehl, J. Lemke and E.W. Mielke, Two lectures on fermions and gravity, In: *Geometry and Theoretical Physics*, Debrus J and Hirshfeld A C (eds.) (Springer, Berlin) p. 56 (1991).
- [36] B. Mashhoon, Length measurement in accelerated systems, Ann. der Physik, **514** 532 (2002).
- [37] B. Mashhoon, Necessity of acceleration-induced nonlocality, Ann. der Physik, **523** 226 (2011); ibidem **520** 705 (2008); arXiv:hep-th/0507157; ibidem **12** 586 (2003); arXiv:0309124[hep-th]; ibidem **198** 9 (1995).
- [38] B. Mashhoon, Limitations of spacetime measurements, Phys. Lett. A, **143**, pp.176-182 (1990).
- [39] B. Mashhoon, The hypothesis of locality in relativistic physics, Phys. Lett. A, **145** pp.147-153 (1990).
- [40] B. Mashhoon, Neutron Interferometry in a Rotating Frame of Reference, Phys. Rev. Lett., **61** pp.2639-2642 (1988).
- [41] B. Mashhoon, On the coupling of intrinsic spin with the rotation of the earth, Phys. Rev. Lett., **198** pp.9-13 (1995).
- [42] J.W. Maluf, F.F. Faria and S.C. Ulhoa, On reference frames in spacetime and gravitational energy in freely falling frames, Class. Quantum Grav., **24** pp.2743-2753 (2007); arXiv:0704.0986[gr-qc].
- [43] J.W. Maluf and F.F. Faria, On the construction of Fermi-Walker transported frames, Ann. der Physik, **520** pp.326-335 (2008); arXiv:0804.2502[gr-qc].
- [44] K.-P. Marzlin, What is the reference frame of an accelerated, Phys. Lett. A, **215** pp. 1-6 (1996).
- [45] W.-T. Ni, On the Proper Reference Frame and Local Coordinates of an Accelerated Observer in Special Relativity, Chinese J. Phys., **15** pp.51-55 (1977).
- [46] W.-Q. Li and W.-T. Ni, On an Accelerated Observer with Rotating Tetrad in Special Relativity, Chinese J. Phys. **16** 214 (1978).
- [47] W.-T. Ni and M. Zimmermann, Inertial and gravitational effects in the proper reference frame of an accelerated, rotating observer, Phys. Rev. D, **17** 1473 (1978).
- [48] F. Gronwald and F.W. Hehl, On the Gauge Aspects of Gravity, Proc. of the 14th Course of the School of Cosmology and Gravitation on Quantum Gravity, held at Erice, Italy, Eds. Bergmann P G, de Sabbata V, and Treder H.-J., World Scientific, Singapore (1996); arXiv:9602013[gr-qc].
- [49] S. Weinberg, Gravitation and Cosmology, J. W. and Sons, New York (1972).
- [50] N.J. Poplawski, Spacetime and fields, pp.1-114 (2009); [gr-qc/0911.0334].
- [51] A. Papapetrou, Einstein's Theory of Gravitation and Flat Space, Proc. Roy. Irish Acad. A, **52** pp.11-23 (1948); Proc.R.Soc. A, **202** 248 (1951); Lectures on general relativity, (Reidel D)(ISBN 9027705402) (1974).
- [52] P.G. Bergmann and R. Thompson, Spin and angular momentum in general relativity, Phys. Rev. **89** 400 (1953).
- [53] C. Møller, Ann. Phys. (NY), On the Localization of the Energy of a Physical System in the General Theory of Relativity, **4** pp.347-371 (1958).